Futures Market Liquidity and the Trading Cost of Trend Following Strategies*

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Abstract

We use a unique dataset reporting the trading of an institutional asset manager implementing trend following strategies to estimate the associated transaction costs. With information both at the trade and the fund levels, we disentangle the sources of these costs by differentiating individual execution quality from managerial decisions. We show that the disappointing performances observed for trend following these recent years are explained by a drop in the volatility of the futures markets these strategies generally trade due to an higher proportion of fixed costs.

1 Introduction

Trend Following strategies are widely used by investors as a source of performance and diversification for their portfolios. These strategies are now well studied by the financial academic literature, and a range of theoretical explanations are provided to better understand the underlying economic mechanisms. However, their success raises important questions regarding the scalability of such strategies. As they involve particularly high turnover and generate significant transaction costs, their attractiveness for investors may strongly depend on the market impact resulting from their trading.

In this paper, we use a unique dataset reporting the trading of an institutional asset manager implementing such diversified trend following strategies on a wide set of futures markets. We use information both at the trade level and at the fund level to cover all the investment decisions that ensure clients an optimal exposure to these strategies. We argue in particular that the information at the trade level alone is not sufficient to reach this objective. Indeed, decisions at the fund level may have a huge impact on the amount of capital traded every day, and by direct consequence on the transaction costs paid by investors. Our approach first consists in computing an implementation shortfall indicator for all trades that are generated by the strategy. This indicator basically measures the difference between the market price used to generate a buy/sell signal, and the average price obtained from all trades related to this signal. The bigger the implementation shortfall is, the higher the transaction cost at the individual trade level. However, this indicator only gives a partial view on the final transaction costs borne by investors. Indeed, all these costs must be aggregated at the fund level. This aggregation process is directly impacted by allocation decisions, for example the decision to increase or not the fund leverage.

Although our approach is limited to a single asset manager and a single investment strategy, it takes into account both the trade-wise and portfolio-wise levels of transaction costs. The alternative approach consists in using bigger databases, such as the one provided by ANcerno. However, this requires a matching procedure to link the trades generated by a strategy to actual trades from the database, to estimate the transaction costs. In our case, we do not have to define such matching procedure, since we possess all the information regarding the reasons why trades are generated.

Our first result is related to the net-of-fee performance of trend following strategies. Using all the information concerning trade execution, we decompose the impact of transaction costs on the performance, at the individual trade level and also at the fund level. We can then evaluate the specific trading costs of each futures market used by the fund. We observe a huge heterogeneity between markets. Moreover, the ranking differs from the one generally obtained when using other transaction cost measures such as the bid-ask spread. Our second result is related to the evolution over time of the total transaction costs paid by investors for trend following strategies. We find that a decrease in the volatility level has a negative impact on these transaction costs. This counterintuitive result can be explained by the adjustment made at the fund level when volatility drops. To maintain the volatility target defined at the fund level, the portfolio manager naturally increases the fund leverage, and by consequence the turnover. If the volatility decrease reduces on average the market impact at the trade level, it increases the total amount of costs paid by investors at the fund level. This result can only be obtained using our dataset, as we get in addition to the information concerning trade-by-trade transaction costs, all the management decisions taken at the fund level to satisfy the fund investment guidelines. Finally, our third result relates to the influence of market conditions on transaction costs. Beyond the volatility drop, the observed drop in the average correlation between markets during the period of 2016-2018 explains the increase in the total transaction costs. Lastly, we show that specific features of trend following strategies may also have an influence on the total transaction costs.

This paper relates to the literature on time-series momentum performance. Jegadeesh and Titman (1993) [17] is the first paper focusing on the momentum anomaly. It shows that past winners tend to outperform past losers in terms of risk-adjusted returns. In other words, a portfolio long the top decile of the past winners and short the bottom decile harvests alpha on the long run, as measured by the standard Fama-French (1992) [12] factor model. Carhart (1997) [8] extends this work in terms of asset pricing by adding this new momentum risk factor to the Fama, French (1992) [12] model. The economic rationale behind the existence of this anomaly lies in behavioural theory. Indeed, according to its founders Kahneman and Tversky (1979) [18], market participants are not perfectly rational and often take decisions outside the scope of the expected utility maximization framework. These decisions are the result of behavioural biases, and some examples are under-reaction, herding, anchoring, disposition effect... Hurst, Ooi and Pedersen (2013) [16] provides a complete review of these biases and explain how these are creating trends in equity markets.

However, this anomaly is present not only in the equity markets but across all asset classes as well (Moskowitz, Ooi and Pedersen, 2012 [23], Baltas and Kosowski, 2012 [3]). The financial industry is well aware of the existence of such anomaly, and trend followers, a special kind of CTAs/Managed Futures from the hedge fund space, have been earning strong returns for decades. The past four years have proven difficult for this style and clients as well as managers are wondering about the future of this strategy and want to understand the reasons behind this recent underformance.[1] Our study contributes to this literature by showing that trading costs may explain part of the disappointing performance observed for these strategies. The drop observed in the market volatility has increased the fund turnover and significantly reduced the performance given to investors.

Moreover, this paper relates to the literature on Transaction Cost Analysis (TCA). The first level of decomposition of transaction costs consists in differentiating explicit from implicit costs. Explicit costs are the ones charged by the broker, the clearer and the exchange to trade, and is a fixed amount per unit of traded quantity. Not much analysis can be done on this matter. Implicit costs are more complex to measure and may depend on many parameters. Indeed, they correspond to the costs of not exactly owning the strategy portfolio, either due to missed trades or to trades realized at a different price than the model price. A first solution to estimate transaction costs is to use the procedure developed in Harris (1989) [15], which consists in estimating the effective bid-ask spread from end-of-day data. Novy-Marx and Velikov (2016) [24] and Chen and Velikov (2019) [9] use this approach to estimate trading costs of a large panel of anomalies including the Carhart momentum. However, this estimate can be quite different from effective transaction costs as it does not account for market impact, i.e. the up/down pressure on prices when we buy/sell. Perold (1988) [25] introduces in his seminal paper the implementation shortfall approach, which decomposes the slippage into different sources of cost, such as delay, bid-ask spread, arrival cost, opportunity cost, market impact... Collins (1991) [11], Wagner (1993) [26], Kissell (2006) [20] and Khandoker (2016) [19] all provide extensions to this decomposition, but the overall idea remains the same. We use the same approach to estimate transaction costs at the individual trade level. These asset-level views of liquidity are interesting when studying how financial markets work. However, from an investment management perspective, the total cost of liquidity should be measured at the

[1] The average annual return of the Société Générale Trend Index is close to -4% the last three years, compared to a previous long-term average close to 10-14%.

portfolio level. Indeed, managing a portfolio comes logically with trading costs, which reduce the achievable expected return. Both stakeholders, manager and clients, have an interest in monitoring them and assessing the trade-off between the expected return and the trading cost. Frazzini, Israel and Moskowitz (2015) [13] use the proprietary database of AQR asset management trades and find lower transaction costs for institutional investors. Two reasons may explain this result. First, the price impact models employed are generally too conservative and overestimate transaction costs. Second, tick-by-tick database provides average trades, including informed traders, retail traders, liquidity demanders, and those facing high price impact costs. By only considering trades executed by institutional investors, they estimate the real costs paid by their clients. Briere et al. (2019) [7] results on ANcerno database confirm Frazzini, Israel and Moskowitz (2015) [13] findings. Our paper contributes to this literature by including in the estimation of transaction costs all the consequences of the fund management decisions at the trade level, with the market impact generated by the rebalancing procedure, but also at the allocation level, when the management constraints evolve with the financial environment. We obtain the counterintuitive result that a drop in volatility can increase the total amount of transaction costs paid to get an exposure to trend following strategies.

The paper proceeds as follows. Section 2 details the characteristics of the fund management process and the methodology used to quantify transaction costs. Section 3 presents the database and the first descriptive statistics illustrating this dataset. Section 4 reports the empirical application. Section 5 concludes.

2 Trade execution quality and fund allocation decisions

The implementation cost of a portfolio or a strategy is simply defined as the cost paid by the manager for the whole associated trading activity. To comply with the fund investment objectives, the fund manager makes transactions with the capital he is entrusted with by clients, plus or minus some additional capital used to leverage/deleverage the portfolio to meet investment objectives. Transaction costs can hence be computed both at the trade (i.e. before leverage) level or at the fund (i.e. after leverage) level. The first level reflects the execution quality per trade, defined by the slippage between the executed price and the target price. The second level reflects the result of the aggregation of these costs, weighed by the transaction size, and may be influenced by general market conditions. We detail in this section the approaches used to quantify both of them.

2.1 Trade execution

Implementation cost is first coming from the execution of individual orders. The corresponding trades are generally executed by algorithms, with the objective to reduce or even reach negative transaction costs. Indeed, the objective of the execution algorithm is to trade all required quantities for the best possible price, that is the closest to the model price or inferior (respectively superior) for a buy (resp. sell) transaction. To do so, the algorithms places limit orders on the supply (resp. demand) side of the order book, hoping for a move in the mid-price to get executed. Should this happen, the obtained price would be better than the model price and the transaction cost negative. However, this trading strategy is not without risk. A move in the opposite direction would leave the limit orders far from the market and new limit orders or market orders should be set. In that case, the executed price would be worse than the model price.

To better understand this idea, let us detail the practical execution of an order. After receiving the trading instructions, the execution algorithm is launched, and this time is stamped as the time of arrival. The mid-price on the market at this time is called the arrival price. During the trading session, the algorithm slices the original order into smaller orders, also called child orders, that may be market or limit orders. Slicing improves the access to liquidity. Indeed, the market impact of child orders is supposedly reduced due to their smaller quantity. Parent orders are not only sliced into child orders, but these child orders are often spread throughout the trading session. This feature, called order scheduling, helps taking a hidden liquidity. However, there is the risk that child orders are not filled at the end of the day. The fill rate, percentage of filled orders over a day, can be used as a measure of execution quality. Anand et al. (2019) [1] analyze broker's routing behavior and performance through this lens. These non-filled orders can be detrimental to the performance, creating an opportunity cost modeled as the difference between the day closing price and a benchmark price. The information contained in our execution set is at the child order level. We focus on the difference between the executed price and the model price, and the latter is necessarily constant across all the child orders related to a parent order. The consequences of slicing and scheduling should be considered in the cost calculation. Our unit and reference for the model is the parent order, where the average executed price is compared to the model price, defined at the beginning of the trading session. The parent order contains the aggregated quantity across all the child orders, and the corresponding average executed price.

2.2 Trade execution quality measure

We estimate trend following trading costs by first computing the cost of rebalancing each futures position and then by aggregating them with their respective weights in the portfolio. The total trading cost on each futures is measured as the sum of the implementation shortfall and fixed costs, including commissions taken from the broker, clearer and the exchange. Implementation shortfall, as defined in Perold (1988) [25], measures the difference between a benchmark price and the average traded price, in percent of the benchmark price. The benchmark price can be the price at the time the strategy desired holdings are generated, or the price observed when the trading session starts. In the first case, the calculation includes the delay cost while in the second case, it measures the sole market impact.

For a parent order m of size Q_m and side S_m (1 for a buy order and -1 for a sell order) composed of N_m child orders, with a respective quantity $q_i, i \in \{1, \ldots, N_m\}$ and executed at a price p_i, the implementation shortfall is given by Equation 1:

$$\mathrm{IS}_m = \frac{S_m}{P_m^{\mathrm{ref}}} \left(\frac{\sum_{i=1}^{N_m} p_i q_i}{Q_m} - P_m^{\mathrm{ref}} \right), \tag{1}$$

where P_m^{ref} is the price used as reference to compute the implementation shortfall. A major difference we have with the standard implementation shortfall from Perold (1988) is the absence of opportunity cost, which is the cost of not trading. We are in a 'complete execution' situation. All orders are always executed before the settlement of the market. Hence, we have the exact decomposition of quantities: $Q_m = \sum_{i=1}^{N_m} q_i$. In that case, the implementation shortfall does not depend on the settlement price of the day. The chosen convention is the standard one, that is to express the Implementation Shortfall as a cost. A buy order ($S_m = 1$) is costly when the average executed price is above the reference price, which is consistent with our formulation. The implementation shortfall can simplify into:

$$\mathrm{IS}_m = \frac{S_m}{P_m^{\mathrm{ref}}} \left(P_m^{\mathrm{exec}} - P_m^{\mathrm{ref}} \right), \tag{2}$$

where P_m^{exec} is the average execution price of the parent order m. Recent papers studying portfolio transaction costs such as Frazzini et al. (2018) [14] and Briere et al. (2019) [7] also use this implementation shortfall approach. The reference price is a key element of the transaction cost analysis, since the interpretation of this cost will be different depending on the chosen reference price. In our case, we consider the model price as the reference so that the implementation shortfall will stand as the difference between paper trading and live strategies. [2]

A parent order is defined as a pair of a contract and a trading day. This parent order thus contains all the child orders across all portfolios that were executed during this day and on a given contract. These child orders are the result of the execution trader slicing and scheduling the parent order to avoid sending one large order which might move the market and make his order badly filled. However, despite this precaution, there still might be information flow during the trading day creating market impact.

Here, since we have all the information about the composition of the parent order (child level information), we can focus on the evolution of the implementation shortfall within the parent order, on an intraday basis. The mathematics stay almost the same, except for the fact that the average execution price of the parent order is updated during the trading session as child orders get executed. The reference price also stays the same, since it is determined before the trading session starts. We accordingly extend Equation 1 to get the intraday implementation shortfall for the k-th child order:

$$\text{IS}_{m,k} = \frac{S_m}{P_m^{\text{ref}}} \left(\frac{\sum_{i=1}^{k} p_i q_i}{\sum_{i=1}^{k} q_i} - P_m^{\text{ref}} \right), \tag{3}$$

with $k \in \{1, \ldots, N_m\}$. The last intraday implementation shortfall necessarily equals the parent order level implementation shortfall, i.e. $\text{IS}_{m,N_m} = \text{IS}_m$. The intraday implementation shortfall as we define it is a partial implementation shortfall since it is the experienced cost of trading the first k child orders. If we consider an advantageous price move between the start of the day and the first child order, the first partial implementation shortfall will be negative and the following partial ones will evolve as new trades are executed but still depend on the preceding child orders execution.

[2]The reference price may be fixed at the previous day closing price. In that case, movements of price happening between the reference time and the start of the trading session would impact the cost. This approach does not produce a cost that makes sense in the execution setup of the analyzed asset management firm.

2.3 Fund allocation decisions

Diversification effects between traded markets and strategies within a fund may reduce the fund volatility. To comply with the fund mandate which often involves a target volatility, the portfolio manager thus needs to leverage up the individual positions. The quantities to trade increase and so does the portfolio implementation cost, assuming a fixed trade execution quality. The Implementation Shortfall measures the relative difference between the executed price and the reference price, and the cost in dollar terms is only the difference between these two prices. On a given day, the portfolio cost is thus the sum of all the dollar costs across all corresponding parent orders, taking into account the quantity traded. We define this cost as:

$$\begin{aligned}\text{Portfolio Trading Cost}_t^{USD} &= \sum_{m \in t} Q_m \times S_m \times \left(P_m^{\text{exec}} - P_m^{\text{ref}}\right) \\ &= \sum_{m \in t} Q_m \times \text{IS}_m \times P_m^{\text{ref}}\end{aligned} \tag{4}$$

where IS_m is the implementation shortfall defined in Equation 1. This is thus the dollar amount lost due to trade on a different price than the model price. It is natural to express it as a performance, by dividing it by the total value of the portfolio. In our firm-wide setup, this corresponds to the Assets Under Management (AUM). The portfolio trading cost on a given day, expressed as a performance, is the following:

$$\text{Portfolio Trading Cost}_t = \frac{\text{Portfolio Trading Cost}_t^{USD}}{\text{AUM}_{t-1}}. \tag{5}$$

Other costs that come from managing a portfolio and placing trades are explicit costs. In the futures markets, these are taken by three different counterparts: the broker for executing each trade, the clearer for ensuring the settlement and the exchange which charges every transaction made. All three are expressed as a fixed cost: a dollar amount is paid for every lot traded. For the purpose of this study, we aggregate these three sources into what we call fixed costs. The portfolio fixed cost Portfolio Fixed Cost_t^{USD}, expressed in dollar terms, is:

$$\sum_{m \in t} Q_m \times \left(\text{BrokerageFee}_t + \text{ClearingFee}_t + \text{ExchangeFee}_t\right), \quad (6)$$

and, expressed as a performance, corresponds to:

$$\text{Portfolio Fixed Cost}_t = \frac{\text{Portfolio Fixed Cost}_t^{USD}}{\text{AUM}_{t-1}}. \quad (7)$$

The portfolio total cost is defined as the sum of these two costs, i.e. the portfolio trading cost and the portfolio fixed cost.

3 Data

We obtain institutional trading data for the period from January 2010 to December 2018 from a CTA management firm. The core trend following strategies used by this fund manager are based on time-series momentum, which is a simple long/short cross-asset portfolio harvesting trends present in futures markets (see Moskowitz Ooi and Pedersen, 2012 [23] for a general discussion on the robustness of the time-series predictability in futures markets). The investment portfolio is basically long the futures which exhibit a past positive performance and short the ones with a negative past return. The position on any contract only depends on its past prices, and contrary to the original momentum, no cross-section comparison is done. Futures are then given an equal budget of risk and the global portfolio targets a fixed volatility. All portfolios supervised by the fund manager follow the same strategy, although with different investment sets or volatility levels. However, the output of the trading system stays the same, a list of trades on futures contracts. Portfolios are rebalanced at a given frequency. Each time, the system sends a list of trades to perform by the execution algorithm, containing desired quantities and model prices. The trading algorithm does not make any explicit portfolio decision, its objective is only to lower the transaction costs related to slippage.

Our data thus reflects the potential costs faced by investors. The context and the scope of our data are very similar to the paper from Frazzini, Israel and Moskowitz (2018) [14]. However, we analyze global futures instead of single stocks markets. Raw data contains all the trades executed by the fund management firm. All portfolios, funds or managed accounts, are managed under the same trend following core system, though some may have investment constraints. Moreover, the portfolios may exhibit different target volatility levels, but this is not a problem since quantities move linearly with the leverage (and so with the target volatility). This execution dataset contains all the trades generated to implement the strategies, named parent orders, and the key characteristics of all child orders, such as an identifier, a date, the underlying futures, the quantity, the price, the broker it was sent to and the clearer. Thanks to the parent order identifier, we relate these trades to other relevant data, necessary to perform our trading cost analysis. Referential data contain the static characteristics of each futures, such as tick size, figure, the related market, the asset class. We also use information relative to the trading session, the model price, the used algorithm. The FX rate is given by the data provider and is necessary to calculate a global consistent price. Finally, some information coming from the firm middle office (such as AUM, broker, clearer and exchange fees) are gathered as well.

	# of Parent Orders ('000)	Total Traded Size ($M)	Avg. Parent Size ($M)	% Buy Orders	Avg. # of child orders per parent order	Avg. Child Size ($M)
Overall	67.92	882 975.80	14.64	53.05	21.72	0.68
2010	0.23	314.60	1.36	46.55	1.08	1.25
2011	2.48	1 748.82	0.71	49.92	1.44	0.49
2012	3.41	2 927.23	0.86	53.82	1.81	0.47
2013	4.65	7 006.57	1.46	51.11	2.40	0.61
2014	6.25	37 401.25	5.93	57.30	7.98	0.74
2015	9.21	57 746.85	6.27	53.32	13.51	0.46
2016	9.91	65 350.68	6.59	51.86	12.66	0.52
2017	11.19	265 292.51	23.70	53.37	32.68	0.73
2018	11.97	388 424.54	32.45	49.14	39.28	0.83

Table 1. Descriptive statistics about the structure of the parent orders in our dataset and its evolution over the years. *Note: Total traded size is the notional value of all traded parent orders, expressed in million dollars. The average parent size is the average notional value of the parent orders, expressed in million dollars. Each parent order is given a direction, which is common to all its child orders, and this information is defined at the parent order level. Each parent order can be sliced in child orders, so the average number of child orders and average size of each are two aspects of this slicing step and can be calculated.*

Tables 1 and 2 describe the parent orders, and decompose them across respectively time and space (asset classes). From a global point of view, the database contains around seventy thousand parent orders and 1.5 million child orders, spanned over nine years. The increase in the number of parent orders is in theory independant from the AUM, since a parent order contains all trades per day on a given futures contract. Each order represents on average close to 15 million dollars,

	# of Parent Orders ('000)	Total Traded Size ($M)	Avg. Parent Size ($M)	% Buy Orders	Avg. # of child orders per parent order	Avg. Child Size ($M)
Overall	67.92	882 975.80	14.64	53.05	21.72	0.68
Agriculturals	6.94	4 994.97	0.79	48.13	15.06	0.05
Bonds	7.78	207 965.98	30.85	52.42	25.93	1.19
Equities	9.74	27 843.68	3.27	55.18	11.59	0.28
Energies	3.33	6 714.68	2.26	50.87	17.86	0.13
Metals	3.30	6 534.79	2.21	49.00	12.33	0.18
Interest Rates	31.53	599 572.28	22.06	54.05	26.14	0.84
Currencies	6.07	29 349.46	5.48	47.85	24.50	0.22

Table 2. Descriptive statistics about the structure of the parent orders in our dataset and its decomposition across asset classes. *Note: Total traded size is the notional value of all traded parent orders, expressed in million dollars. The average parent size is the average notional value of the parent orders, expressed in million dollars. Each parent order is given a direction, which is common to all its child orders, and this information is defined at the parent order level. Each parent order can be sliced in child orders, so the average number of child orders and average size of each are two aspects of this slicing step and can be calculated.*

though with a large time variation: from around 1 million in the first years to around 25-30 million dollars for the last three years. The first factor behind this is the increase in the AUM, which all things equal increase the quantity of all orders. The second factor is the leverage used by the manager to target the fund volatility. The increase in the number of child orders per parent, which measures the slicing intensity, is strong and close to the one observed on the average parent size. It is confirmed by a relatively stable size of child orders. The percentage of buy orders stays close to 50% during the whole period.

From Table 2, interest rates futures are by far the most traded in terms of number of parent orders, and dollar value. The number is around ten times higher than the ones observed in energies or metals. This huge discrepancy has to be kept in mind when looking at the portfolio-level trading cost. It comes from the low volatility level observed in rates futures markets. Any significant position on these markets implies a high average parent order size. In addition, it seems that interest rates, bonds and currencies all exhibit a higher number of child orders per parent. This slicing feature of the algorithm on these asset classes matches the fact these markets have deep limit order books, which quickly replenish themselves. Liquidity aspects of each futures market can be taken into account in the execution algorithm. The buy percentage remains close to 50% across asset classes, with a small long bias on equity and interest rates markets.

Tables 3 and 4 describe the size of parent orders, expressed as a traded quantity, and their

	Avg. Qty (#)	Avg. Volume (#)	Avg. Open Interest (OI) (#)	Avg. POV (bps)	Avg. POOI (bps)
2010	7.18	231 437.29	334 749.21	1.34	0.37
2011	3.83	209 969.67	559 350.44	2.09	0.20
2012	4.78	150 081.92	464 035.75	0.65	0.22
2013	7.48	171 005.67	498 429.28	1.13	0.27
2014	29.93	172 343.45	532 193.94	4.62	1.04
2015	35.27	170 689.21	540 680.32	5.83	1.28
2016	41.65	189 345.20	567 290.27	5.52	1.64
2017	136.20	184 968.56	620 821.77	18.16	4.73
2018	170.33	219 561.74	709 916.76	19.52	4.94

Table 3. Size of parent orders and their participation rate in futures markets, through time. *Note: Quantity is expressed as the number of lots of each parent order, and is compared to the daily market volume and open interest observed on that day. Then, values are averaged across parent orders. POV stands for Pourcentage of Volume, and POOI for Pourcentage of Open Interest. The POV (respectively POOI) is the ratio of the parent order total absolute quantity to the daily volume (resp. open interest) of the corresponding day.*

participation rates in corresponding futures markets. About the evolution through time, parents orders have gone bigger, essentially due to the increase in the assets under management (AUM). However, it is difficult at this stage to interpret this evolution over time, as the average depends on the portfolio composition at each date. Indeed, the stated average volume is the average of the daily volume across all parent orders, so depend on the the markets traded on the corresponding period. The same goes for the average open interest. Only the calculations reported on Table 4 do not depend on the asset class weights within the portfolio.

The increase in the traded quantities seems more important than the one of the volume and open interest. Values of participation rates are relatively small, both considering volume and open interest. Indeed, the total trading of the asset manager only represents up to 20 basis points of the daily volume, with an overall overage of 11 basis points.

In Table 4, the asset class decomposition gives some insights regarding the futures market liquidity. Bonds and equities appear to be the most liquid ones, followed by interest rates which display a small volume in comparison to the open interest. The first asset class where futures trade, the agriculturals markets, exhibits also this phenomenon. Both volume and open interest on currency futures are low, which appear as a surprise, since this asset class is the largest one in terms of total dollar volume across all currency contracts. This may be the result of a large portion of investors using forwards instead of futures. The participation rates of the trading of the studied

	Avg. Qty (#)	Avg. Daily Volume (#)	Avg. Daily Open Interest (OI) (#)	Avg. POV (bps)	Avg. POOI (bps)
Agriculturals	29.60	64 652.56	236 469.03	7.26	2.40
Bonds	214.67	539 580.56	1 400 515.11	5.78	2.91
Equities	33.95	332 532.72	704 505.31	5.31	2.30
Energies	40.33	181 607.61	200 215.11	4.66	3.26
Metals	24.81	84 645.65	132 643.10	5.90	3.32
Interest Rates	89.84	117 417.84	633 842.44	15.43	1.91
Currencies	71.87	102 482.32	172 568.26	10.75	6.35

Table 4. Size of parent orders and their participation rate in futures markets, across asset classes. *Note: Quantity is expressed as the number of lots of each parent order, and is compared to the daily market volume and open interest observed on that day. Then, values are averaged across parent orders. POV stands for Pourcentage of Volume, and POOI for Pourcentage of Open Interest. The POV (respectively POOI) is the ratio of the parent order total absolute quantity to the daily volume (resp. open interest) of the corresponding day.*

money manager remain low across all asset classes.

4 Results

We present in this section all the results obtained both at the individual trade level and at the fund level, i.e. including all the allocation decisions made by the manager to respect the funds' investment policy.

4.1 Execution costs at the trade level

Tables 5 and 6 report the Implementation Shortfall (IS) across time between 2012 and 2018 and across asset classes, along with standard proxys of liquidity such as tick size and volatility. Indeed, Kyle (1985) [22] (see also Bacry et al., 2015 [2], and Frazzini, Israel and Moskowitz, 2018 [14]) consider bid-ask spread and volatility as factors that explain market impact. [3] A first observation is related to the overall average implementation shortfall, which is of 1.29 basis points. This is much lower than what is observed in the equity cash market, where the implementation shortfall is around 15 basis points (as confirmed by Briere et al, 2019 [7] and Frazzini et al, 2018 [14]). Over the years, the average IS varies between 1.02 and 2.10 basis points. The median appears in

[3]Results on IS for 2010 and 2011 are not reported as the number of trades is too small to ensure economic significativity.

	Avg. IS (bps)	Median IS (bps)	Avg. IS Sell (bps)	Avg. IS Buy (bps)	Avg. Tick Size (bps)	Avg. Annual Vol. (%)
Overall	1.29	0.34	1.38	1.21	4.03	7.97
2012	1.73	0.25	-0.09	3.29	3.45	7.65
2013	1.48	0.45	1.76	1.22	4.34	7.06
2014	1.51	0.46	1.80	1.30	4.32	6.21
2015	2.10	0.50	3.20	1.18	4.65	8.01
2016	1.03	0.34	1.80	0.36	4.01	9.29
2017	1.02	0.25	0.80	1.20	3.54	7.82
2018	1.47	0.34	1.63	1.30	3.70	7.46

Table 5. Yearly summary of the trade level Implementation Shortfall and related liquidity characteristics. *Note: Tick Size is calculated as the ratio of the tick to the parent order reference price and annual volatility corresponds to the volatility estimated over the year before the trade date of the corresponding parent order (for the corresponding futures market). Both averages are done across all relevant parent orders.*

	Avg. IS (bps)	Median IS (bps)	Avg. IS Sell (bps)	Avg. IS Buy (bps)	Avg. Tick Size (bps)	Avg. Annual Vol. (%)
Overall	1.29	0.34	1.38	1.21	4.03	7.97
Agriculturals	4.62	3.82	4.38	4.88	3.93	21.50
Bonds	0.72	0.45	0.63	0.80	0.89	3.74
Equities	0.78	0.99	2.36	-0.49	1.09	14.46
Energies	3.26	2.79	0.94	5.48	1.60	29.84
Metals	4.30	3.25	4.20	4.40	1.47	19.35
Interest Rates	0.36	0.26	0.33	0.39	0.60	0.18
Currencies	0.99	0.70	0.78	1.22	33.06	9.06

Table 6. Asset class summary of the trade level Implementation Shortfall and related liquidity characteristics. *Note: Tick Size is calculated as the ratio of the tick to the parent order reference price and annual volatility corresponds to the volatility estimated over the year before the trade date of the corresponding parent order (for the corresponding futures market). Both averages are done across all relevant parent orders.*

every case except 2010 inferior to the mean, which is a sign of positive skewness: there are parent orders with very large implementation shortfalls. No specific pattern appears between the buy versus short side when comparing the average IS. Concerning the last three years of our sample, i.e. the period 2016-2018, we observe no specific upward trend in transaction costs, while trend following strategies exhibited negative performances. The result reported in Table 5 is the first confirmation that this disappointing performance is not related to a deterioration of the execution capability at the trade level. The use of IS allows to decompose the trading and zoom in on the trade level to obtain this result. The ability of the fund to access the futures market liquidity is not deteriorated even if the size of the fund increases a lot during the period. Concerning the evolution of the average volatility, we observe this time a decrease, from 9.29% in 2016 to 7.46% in 2018. Considering equal quantities across asset classes, a decrease in volatility should be associated to a decrease in the implementation shortfall. Tables 11 and 12 in Appendix contain extremes quantiles of the IS distribution, through time and across asset classes. The 5% quantile is on every case below the 95% in absolute terms, confirming the right skewness.

We show in Table 6 that as expected, the implementation shortfall is the highest in commodity futures with a value five to ten times larger than in standard asset classes. This asset class displays both higher volatility and lower volume (on average). Looking at asset class decomposition, the number of parent orders seems to be out of phase from the average IS. These findings suggest the calculation of a quantity-weighed IS, or in other words, the total trading cost at the portfolio level.

4.2 Portfolio-level trading costs

	Portfolio Trading Cost (%)	Portfolio Fixed Cost (%)	Portfolio Total Cost (%)
2012	3.06	0.63	3.69
2013	6.04	1.14	7.18
2014	4.49	0.82	5.31
2015	2.92	0.47	3.39
2016	1.25	0.41	1.66
2017	2.71	0.66	3.37
2018	3.45	0.65	4.10

Table 7. Yearly decomposition of the aggregated (portfolio level) trading costs. *Note: Portfolio Trading Cost is the cost in dollars coming from the difference between the executed price and the model price, divided by the last AUM level to obtain a relative performance measure. Portfolio Fixed Cost is the explicit cost due to the commissions taken by the broker, the clearer and the exchange (also presented as relative to AUM). Portfolio Total Cost is the sum of these two costs.*

Tables 7 and 8 report the total cost of trading, which is the sum of implicit (trading) and explicit (fixed) costs, decomposed across time and asset classes. The time-series variation of the annual trading cost is quite large, with almost a factor of five between the mininum and maximum, whereas the fixed cost is stable, around 70 basis points per year. The average annual total cost of trading is 3.20%. Surprisingly, the evolution of the portfolio trading cost does not match the one of the implementation shortfall. Indeed, high cost does not coincide with high implementation shortfall. The cost borne by the fund manager and the client is not fully reflected by the trade-by-trade implementation shortfall. Indeed, we observe a huge increase in the portfolio total cost

	Portfolio Trading Cost (%)	Portfolio Fixed Cost (%)	Portfolio Total Cost (%)
Agriculturals	1.66	0.39	2.05
Bonds	6.32	0.88	7.20
Equities	2.22	0.49	2.71
Energies	2.20	0.23	2.43
Metals	1.36	0.18	1.54
Interest Rates	11.60	2.69	14.29
Currencies	1.23	0.53	1.76

Table 8. Asset class decomposition of the aggregated (portfolio level) trading costs. *Note: Portfolio Trading Cost is the cost in dollars coming from the difference between the executed price and the model price, divided by the last AUM level to obtain a relative performance measure. Portfolio Fixed Cost is the explicit cost due to the commissions taken by the broker, the clearer and the exchange (also presented as relative to AUM). Portfolio Total Cost is the sum of these two costs.*

between 2016 and 2018 while the IS on the same period remains stable. This result can explain the disappointing performance of trend following strategies in the 2016-2018 period. Part of the performance drop can indeed be explained by this increase of the transaction costs at the fund level, even if the execution quality remains stable. We have then to understand which mechanism is responsible for this phenomenon.

In terms of asset class repartition, we observe on Table 8 a similar disconnection between the individual level cost and the portfolio level cost. Agriculturals and metals have a high implementation shortfall on average, but are the sectors which contribute the less to the portfolio cost. Interest rates exhibit an average IS of 36 basis points (per trade) but is by far the largest contributor at the portfolio level, with a total cost of 11.60%. An indication of this difference comes from the fixed costs, due to their direct relation with the traded quantity. Commissions are different between futures, but overall this portfolio fixed cost increases as the traded quantity increases. The huge difference between interest rates and other asset classes fixed costs might be strongly explained by a difference in turnover.

4.3 Transaction costs, turnover and market conditions

The objective in this section is to explain the increase in the total trading cost of trend following, which essentially happened during the last three years. We include now all the information we have on the management decisions taken at the fund level. In particular, the leverage used to meet volatility objectives has an impact on the fund turnover and the transaction costs.

Turnover is the sum of traded quantities. The effect of the amount under management is linear with the target positions, and so on the turnover. A way to control for that is to calculate the turnover per million dollar traded, which is a relative measure of turnover. This feature of the trading seems to explain the time-series variation of the portfolio total trading cost, especially for the last three years. However, this correction is not perfect. Assuming the new money that recently flew into this asset management firm were under the lowest volatility program, the associated traded quantities would be much lower than the same amount managed at a higher volatility target. In other words, a portfolio targeting 20% volatility with 10 million dollars generates the same amount of trading that a portfolio targeting 5% volatility with 40 million dollars. To control for that, the assets under management of each portfolio must be rescaled each day to match a common volatility-equivalent AUM. Please refer to the appendix for the mathematics.

We also consider in this subsection the impact of market conditions on transaction costs. In addition to the average volatility indicator in the previous section, we define an average correlation indicator. A way to measure this "average correlation" between the traded assets is to consider a standard principal component analysis and evaluate the pourcentage of variance explained by the first axis. A high value indicates that all futures markets are actually much related to one factor, which is a linear combination of them. Trend following strategies can be long or short any futures market, so a high negative correlation between futures should count as a high positive correlation. In addition to not being statistically meaningful, an average of the correlations would not capture this long/short issue. Trend following strategies are very often target volatility products. So, when the volatility of individual futures markets decrease, the fund manager mechanically has to increase the leverage to maintain the global exposure. Another potential source of turnover increase is the other factor influencing the portfolio volatility, namely the diversification. Indeed, a correlation decrease will reduce the portfolio volatility, so positions must be increased to reach the target volatility level. Let's look at the simple diversification mathematics. An equally-weighted long-only portfolio of N assets, with equal volatility σ_i and equal correlations ρ, has the following volatility:

$$\sigma_P = \sigma_i \sqrt{\frac{1 + (N-1) \times \rho}{N}} \tag{8}$$

Table 9 contains the total cost of trading, the raw and relative turnover measures, the market

	Portfolio Total Cost (%)	Turnover (#)	Adjusted Turnover ($\frac{\#}{\text{AUM}}$)	Avg. Annual Volatility (%)	Avg. Explained Variance (%)	SG Trend Annual Return (%)
2012	3.69	16 291	3 261.06	7.65	42.30	-3.52
2013	7.18	35 627	5 622.21	7.06	40.45	2.67
2014	5.32	185 065	5 049.62	6.21	26.41	19.70
2015	3.39	319 610	3 778.73	8.01	23.59	0.04
2016	1.66	402 384	3 729.89	9.29	29.15	-6.14
2017	3.37	1 512 247	5 820.92	7.82	27.68	2.20
2018	4.10	2 026 045	5 712.77	7.46	20.42	-8.11

Table 9. Total trading cost evolution and potential related features. *Note: Turnover is the total quantity (number of lots) traded. Adjusted Turnover is the quantity traded per amount of AUM, previously rescaled between portfolios to take into account their different volatility targets. The average explained variance is the yearly average of the pourcentage of variance explained by the first axis of a standard PCA, applied on the futures markets returns, and can be viewed as an "average correlation".*

environment variables and the corresponding performance of the Société Générale Trend Index, which is the standard benchmark for trend following strategies. The turnover, expressed as the total quantity traded over the year, dramatically increases. However, this effect is mainly driven by the increase of the assets under management of the firm.

The phenomenon of volatility and correlations drop appears to be the reason of the increase in the adjusted turnover: the volatility gradually decreased from 9.29% to only 7.46% over these three years. In addition, our "average correlation" indicator exhibits a drastic decrease from 2016 to 2018, going from 29% to around 20%. Along with the drop in the markets' individual volatilities, a reduced correlation lowers the ex ante portfolio volatility, all other things equal. We are able to explain the increase in the adjusted turnover by a decrease in the volatility and in the homogeneity of the markets. We can proxy our dynamic strategy, which is long and short and with time-varying positions, to a long-only portfolio equally-invested across assets with the average volatility (equal weight feature) and pourcentage of explained variance (long-only feature) from Table 9. For 2016, a portfolio of 50 assets would exhibit an annual volatility of 5.02%, whereas in 2018, the same calculation would result in a 3.37 % annual volatility. Assuming a volatility target of 20%, we can imply that positions were leveraged by a factor 4 on average in 2016, whereas a factor 6 was needed to reach the target level in 2018. Finally, this 1.5 factor appears to fit well the variation observed in the adjusted turnover. From this, we can conclude that a very large portion (around 98%) of the increase in the adjusted turnover comes from the joint evolution of volatilities and correlations.

The performances of trend followers, as proxied by the SG Trend Index, were disappointing since 2015 in comparison to the long-term track record of this strategy. The average annual return of around -4% could be the result of the increase of trading costs. However, this is not the result of a worsen quality of trading but rather of an increase in the quantities traded, which itself is due to a decrease in the average volatility and correlations of futures markets.

Table 15 in Appendix gives additional insights regarding the asset-class decomposition of the yearly adjusted turnover, especially the high contribution of interest rates to cost is explained by their large preponderance in the portfolio adjusted turnover. This is also consistent with the interest rates having a much lower volatility than other markets (please refer to Table 16 in Appendix for the average yearly volatility per asset class).

4.4 Turnover and trends

It is well-known that equity momentum strategies generate large turnover, as shown amongst others by Korajczyk and Sadka (2004) [21], Barroso and Santa Clara (2015) [6], Frazzini et al. (2015) [13] and Briere et al. (2019) [7]. Since trend following or time-series momentum is also based on the momentum anomaly, one could argue the latter comes with high turnover as well. However, trend following is directional and trade futures, often less numerous than the single stocks belonging to an index. Both aspects may have an impact on the turnover. Baltas (2015) [4] analyzes the turnover of trend following strategies and finds it is three to fives times larger the one of a comparable long-only strategy. In a companion paper, Baltas and Kosowski (2013) [5] go one step further and study the relationship between turnover and portfolio features such as rebalancing frequency and holding period. The intuition is confirmed: daily reshuffling involves a turnover ten times larger than the one of the monthly reshuffled portfolio. The objective in this section is to explain the evolution of the turnover of trend following strategies by features that are specific to this strategy. We assume here that the turnover depends on the markets' trend quality (which is partly dependant on the lookback period of the strategy) and on their cotrends. A ranging market, which keeps going up and down with very short trends (relative to the lookback horizon), thus misleads the trend follower who switches frequently his position, creating turnover. However, when a trend is continuating, the initial position is usually increased, which creates turnover. This turnover is the result of an improving trend quality at the individual market level. The mechanism through which the correlation between trends impacts the turnover is more difficult to grasp. In general, the portfolio direction (long or short) on a given market only depends on this futures market. However, this position may be reduced or increased if a cotrend risk is taken into account to assess the portfolio global risk, in a similar fashion as the diversification effect (due to the correlations) we documented in the previous section.

Based on Chevalier and Darolles (2019) [10] signature methodology, we extract the *signed* returns that will be used for both assessing the overall trending quality and calculating an average "cotrend" thanks to a standard PCA decomposition, as we did in the previous section.

	Adjusted Turnover ($\frac{\#}{\text{AUM}}$)	Avg. Trend Quality (%)	Avg. Explained *Signed* Variance (%)
2012	3 261.06	14.57	38.79
2013	5 622.21	12.82	39.39
2014	5 049.62	16.90	29.91
2015	3 778.73	22.40	33.61
2016	3 729.89	16.43	32.19
2017	5 820.92	14.45	37.02
2018	5 712.77	14.33	41.40

Table 10. Total trading cost evolution and trends' features. *Note: The average explained Signed variance is the yearly average of the pourcentage of variance explained by the first axis of a standard PCA, applied on the futures markets signed returns, which are obtained by applying the signature methodology on raw returns.*

Table 10 reminds the yearly relative turnover, along with the contemporaneous average of both the trend quality and the pourcentage of explained signed variance, which we call an "average cotrend". It appears the turnover decreases as the quality of trends deteriorates. We can interpret this as the following fact: a trend continuation increases less the turnover than a change in the direction of the trend (long to short and vice-versa). Concerning the other market feature, it seems the more trends are correlated the higher the turnover. Further analysis is needed to better disentangle both effects.

5 Conclusion

With information on both trades and management decisions, we analyze the different sources of transaction costs. We show that the drop in the volatility of futures markets is the main factor explaining the relatively low performances of trend following strategies during the period starting from 2016 to 2018. In addition, specificities of the trend following strategies, such as markets trend quality and trend risk, influence the portfolio construction, and so the resulting turnover. We also show that the classic liquidity measures, such as bid-ask spread, do not give a good view on the effective transaction costs for a given strategy. All our analysis shows that it is essential to include in a transaction cost analysis some information regarding the fund management.

The next step of our analysis would be to use our dataset to calibrate a more general market impact model to allow some capacity calculation without any reference to a particular AUM historical path. This is left for future research.

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6 Appendix

6.1 Implementation Shortfall distribution

	$q_5(IS)$ (bps)	$q_{95}(IS)$ (bps)
Overall	33.63	-28.46
2010	38.63	-47.86
2011	39.13	-40.26
2012	26.86	-20.86
2013	30.83	-22.78
2014	33.40	-23.46
2015	44.93	-39.33
2016	37.40	-31.48
2017	32.35	-28.04
2018	32.41	-26.00

Table 11. Extremes quantiles of the IS distribution, through time.

	$q_5(IS)$ (bps)	$q_{95}(IS)$ (bps)
Overall	33.63	-28.46
Agriculturals	75.17	-65.81
Bonds	13.20	-10.90
Equities	41.10	-39.80
Energies	98.47	-86.62
Metals	56.36	-44.65
Interest Rates	1.51	-0.53
Currencies	25.10	-22.01

Table 12. Extremes quantiles of the IS distribution, across asset classes.

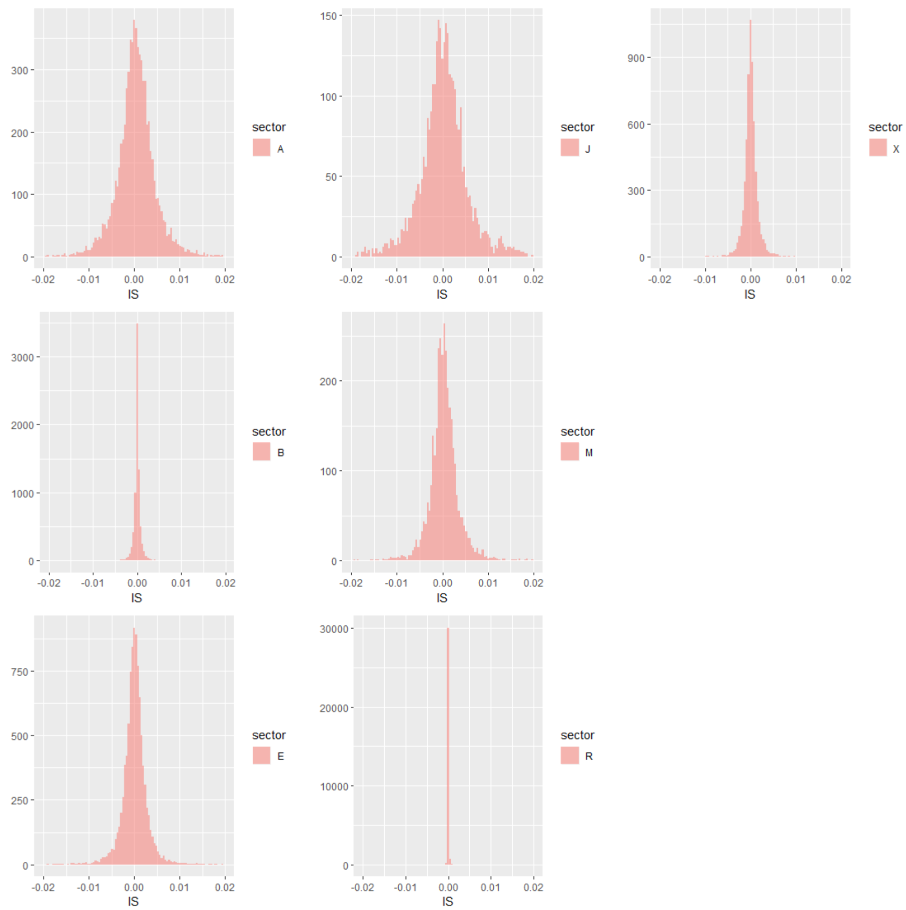

Figure 1. Asset Class distribution of the implementation shortfall.

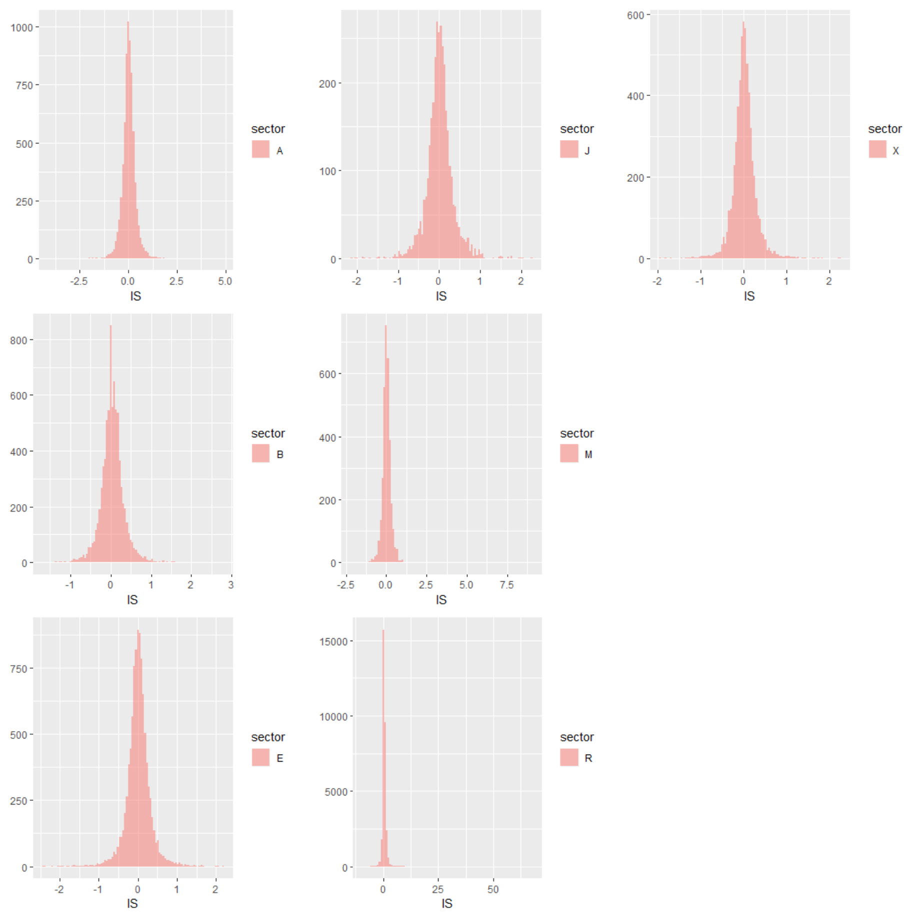

Figure 2. Asset Class distribution of the volatility-adjusted implementation shortfall. *Note: The volatility we use here corresponds to the volatility estimated over the year before the trade date of the corresponding parent order, for the corresponding futures market.*

	Avg. IS (bps)	Avg. Tick-adjusted IS (#ticks)	Avg. Vol (%)	Avg. Vol-adjusted IS (# vol.)	Median IS (bps)	Median Tick-adjusted IS (#ticks)	Median Vol (%)	Median Vol-adjusted IS (# vol.)
2010	-0.45	-0.06	16.22	0.03	0.42	0.02	14.40	0.00
2011	0.24	-0.12	9.28	0.09	0.00	0.00	0.79	0.00
2012	1.73	1.49	7.65	0.11	0.25	0.09	0.46	0.02
2013	1.48	1.54	7.06	0.15	0.45	0.30	0.71	0.03
2014	1.51	1.76	6.21	0.22	0.46	0.50	0.59	0.06
2015	2.10	1.37	8.01	0.28	0.50	0.78	0.32	0.10
2016	1.03	0.88	9.29	0.18	0.34	0.50	1.68	0.05
2017	1.02	0.89	7.82	0.22	0.25	0.33	4.12	0.05
2018	1.47	1.66	7.46	0.22	0.34	0.50	4.00	0.07
2019	0.75	0.93	8.59	0.16	0.34	0.41	5.47	0.05

Table 13. Additional description of the implementation shortfall, per year. *Note: The tick-adjusted IS is the slippage (difference between model price and executed price) expressed as a number of ticks. The vol-adjusted IS is the standard IS, expressed as a number of daily volatilities.*

	Avg. IS (bps)	Avg. Tick-adjusted IS (#ticks)	Avg. Vol (%)	Avg. Vol-adj. IS (# vol.)	Median IS (bps)	Median Tick-adj. IS (#ticks)	Median Vol (%)	Median Vol-adj. IS (# vol.)
Ags.	4.62	1.33	21.50	0.03	3.82	1.00	20.56	0.03
Bonds	0.72	0.78	3.74	0.04	0.45	0.71	3.71	0.04
Equities	0.78	1.03	14.46	0.01	0.99	1.00	13.50	0.01
Energies	3.26	4.02	29.84	0.02	2.79	2.00	28.92	0.02
Metals	4.30	5.32	19.35	0.03	3.25	2.33	18.87	0.03
Rates	0.36	0.70	0.18	0.40	0.26	0.50	0.17	0.25
FX	0.99	0.68	9.06	0.02	0.70	0.02	8.45	0.01

Table 14. Additional description of the implementation shortfall, across asset classes. *Note: The tick-adjusted IS is the slippage (difference between model price and executed price) expressed as a number of ticks. The vol-adjusted IS is the standard IS, expressed as a number of daily volatilities.*

6.2 Focus on costs features

	Ags.	Bonds	Equities	Energies	Metals	Interest Rates	Currencies	Total
2010	12.86	51.64	23.46	13.16	8.36	206.15	22.46	338.10
2011	49.65	364.35	129.50	81.67	39.69	811.33	143.41	1 619.59
2012	109.77	1 100.46	203.35	118.66	59.96	1 416.46	252.39	3 261.06
2013	212.97	1 564.59	366.42	114.96	69.12	2 968.22	325.93	5 622.21
2014	154.85	1 307.77	324.49	120.41	65.57	2 730.21	346.31	5 049.62
2015	95.62	1 257.69	181.45	80.77	46.54	1 855.36	261.30	3 778.73
2016	110.00	1 486.45	224.72	66.95	48.54	1 488.13	305.10	3 729.89
2017	185.01	1 582.03	434.56	126.55	76.10	3 023.87	392.80	5 820.92
2018	204.43	1 699.49	280.93	154.43	90.58	2 881.53	401.37	5 712.77

Table 15. Relative Turnover (#/ million $) decomposed across years and asset classes. *Note: The relative turnover is the yearly sum across all relevant parent orders, of the absolute traded quantity divided by the firm volatility-adjusted AUM.*

	Ags.	Bonds	Equities	Energies	Metals	Interest Rates	Currencies	Overall
2010	28.02	5.30	18.22	31.55	24.68	0.33	11.24	16.22
2011	28.55	4.68	19.21	29.57	26.09	0.32	11.43	9.28
2012	26.22	4.38	19.60	28.93	25.10	0.33	9.88	7.65
2013	20.24	3.79	13.96	23.85	22.19	0.20	8.62	7.12
2014	19.76	3.44	12.02	20.40	17.38	0.15	6.96	6.20
2015	22.07	4.15	16.86	33.14	19.53	0.14	10.45	8.01
2016	22.47	4.42	18.45	41.40	22.25	0.19	11.24	9.29
2017	21.93	3.79	11.32	30.50	19.00	0.15	9.51	7.82
2018	20.45	3.14	11.96	25.32	16.85	0.16	8.06	7.46

Table 16. Average Annualized Volatility decomposed across years and asset classes. *Note: The average volatility is calculated across all relevant parent orders, whose daily volatility is the past one year estimate.*

6.3 Relative Turnover

The raw turnover writes:

$$\text{Turnover}_t = \sum_m Q_m \tag{9}$$

The turnover per million dollars is:

$$\text{Relative Turnover}_t = \sum_m \frac{Q_m}{\text{AUM}_t} \tag{10}$$

where AUM_t is the total assets under management (in dollars) at date t managed by the firm.

Assume the following decomposition across portfolios:

$$\text{AUM}_t = \sum_p \text{AUM}_t^p \tag{11}$$

Each portfolio p is managed at target volatility σ_p. Its actual aum value AUM_t^p is equivalent to the following value of AUM managed at the volatility σ_{ref}: $\text{AUM}_t^p \times \frac{\sigma_p}{\sigma_{\text{ref}}}$. The total rescaled AUM writes:

$$\tilde{\text{AUM}}_t = \sum_p \text{AUM}_t^p \times \frac{\sigma_p}{\sigma_{\text{ref}}} \tag{12}$$

Finally, the volatility-adjusted version of the turnover per million dollars is:

$$\text{Relative Turnover}_t^{\text{ref}} = \sum_m \frac{Q_m}{\tilde{\text{AUM}}_t} \tag{13}$$

6.4 Capacity simulation

6.4.1 Mathematics. The estimated portfolio fixed cost ($\hat{\text{EFC}}$) writes:

$$\begin{aligned}
\hat{\text{EFC}} &= \frac{\sum_m \tilde{Q}_m \times (\text{BrokerageFee} + \text{ClearingFee} + \text{ExchangeFee})}{\text{AUM}_{\text{target}}} \\
&= \frac{\sum_{m \in t} Q_m \times \frac{\text{AUM}_{\text{target}}}{\text{AUM}_m} \times (\text{BrokerageFee} + \text{ClearingFee} + \text{ExchangeFee})}{\text{AUM}_{\text{target}}} \\
&= \frac{\sum_{m \in t} Q_m \times (\text{BrokerageFee} + \text{ClearingFee} + \text{ExchangeFee})}{\text{AUM}_m} \\
&= \text{Portfolio Fixed Cost}
\end{aligned} \tag{14}$$

6.4.2 Approach and results. This section aims at estimating the total trading cost at different levels of assets under management. This simulation would allow to assess the total capacity of a trend following strategy. Instead of estimating the cost through an analysis of parent orders, we will use the composition of the child orders and their partial implementation shortfall to calculate the theoretical cost one would pay when executing a smaller total quantity. Essentially, we are going to estimate a function linking the AUM and the related trading cost in the neighbourhood of the maximum AUM of the management firm. We wil then extend this function for higher values of AUM.

The procedure is as follows. A level of AUM is fixed, $\text{AUM}_{\text{Target}}$, (lower than the maximum observed) and we filter in all parent orders that happened a day where the AUM was higher than the fixed level. The total quantity Q_m is then rescaled to get the total quantity that would have been traded if the AUM was at the target level:

$$\tilde{Q}_m = Q_m \times \frac{\text{AUM}_{\text{Target}}}{\text{AUM}_m}. \tag{15}$$

Due to our filter, we necessarily have $\text{AUM}_{\text{target}} \leq \text{AUM}_m$, so $\tilde{Q}_m \leq Q_m$. Then, for each parent order, we identify the child order that allows to reach a total traded quantity of at least $\tilde{Q}_m$ and we take the corresponding average executed price as the theoretical price one would have got, noted $\tilde{P}_m^{\text{exec}} = \frac{\sum_{i=1}^{k} p_i * q_i}{\sum_{i=1}^{k} q_i}$, with $k = \min\{c / \sum_{j=1}^{c} q_j \geq \tilde{Q}_m\}$.

We can calculate now the trading cost, considering both our theoretical quantity and theoretical fill price. These are called theoretical beacuse of the assumption of a lowered AUM, but parent orders went exactly through these levels at one point during the day so these prices are nonetheless realistic. Then, we aggregate the trading cost across the filtered set of parent orders. To get a yearly total cost, the cost must be rescaled via an estimated of the number of days in a year and the number of days in the filtered set, N_S. The final estimated yearly cost ($\hat{\text{ETC}}$) writes:

$$\hat{\text{ETC}}(\text{AUM}_{\text{Target}}) = \frac{\sum_{m \in S} \tilde{Q}_m \times S_m \times \left(\tilde{P}_m^{\text{exec}} - P_m^{\text{ref}}\right)}{\text{AUM}_{\text{Target}}} \times \frac{N_S}{260}. \tag{16}$$

The portfolio fixed cost should also be reassessed at the theoretical AUM level. However, these commissions are fixed and only depend on the quantity. When writing this cost as a performance, in other terms relative to the AUM level, the linear change in the AUM is balanced by the same change in the quantity, making the ratio constant. To avoid depending on the costs over one year, which is related to the trading of futures over this particular year, we will use the whole sample as a global estimation of the average trading, in terms of futures market repartition. The estimated annual fixed cost is 0.66%.

Figure 3 displays the AUM level we set at the beginning of the procedure and the corresponding trading cost we would have got, had we filled lower trading quantities. In order to predict potential trading cost at larger AUM, we estimated the following linear model thanks to the standard OLS

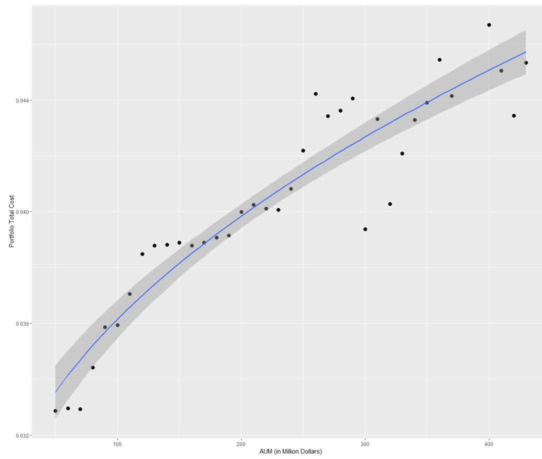

Figure 3. Estimated yearly trading cost and the corresponding theoretical AUM level.

estimator:

$$\text{Cost} = \alpha + \beta \times \sqrt{\text{AUM}} + \epsilon \tag{17}$$

Since there is no cost associated with a null AUM, there should be a non-continuity on the positive side of 0 which would make the model misspecified on this part. Our model does not suffer from that since we start with an AUM of 50 million dollars. We interpret the intercept as the following: it is the cost that comes with executing orders once one starts trading, say with a \$1M AUM. We estimate it at around 2.72%. The beta coefficient of the regression, interpreted as the marginal cost related to managing an additional million dollars, is estimated at 8.92 basis points. The R^2 of the regression is 88.3%. With strong and complex assumptions regarding the liquidity of the markets, the structure of their order books, and the trading turnover of a trend following strategy, we are able to estimate the cost of managing an extra million dollars which gives insights regarding the capacity of these strategies. Trend following strategies, as proxied by the SG Trend Index, exhibit a 0.4 net Sharpe Ratio since 2000. At a target volatility of 20%, it is equivalent to delivering an annual return of 8%. The AUM that would bring the total trading cost to the level of the net-of-fees performance, called breakeven AUM, is \$3 500M.