New stylized facts of financial exuberance periods*

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March 7, 2022

Abstract

Stylized facts of asset returns are widely established. Among these are e.g. non-normality, volatility clustering and high persistence. Another important recurring aspect is the existence of financial exuberance, often interpreted as an explosive price bubble. Exuberance periods consist of two parts, (i) the temporary explosive price period and (ii) the mean-reverting reverse period of market correction. We provide a comprehensive analysis of exuberance periods by analysing 30 markets from different categories over a time period of fifty years. We cover international stock markets, the US housing market, Gold, Silver and Oil as well as the Bitcoin prices. Overall, we find 143 exuberance phases and document evidence on important characteristics like (i) durations of explosive phases, (ii) collapse duration and behaviour during market correction phases, (iii) magnitude of autoregressive parameters during exuberance and market correction and (iv) distributional characteristics like fat tails and shifts in the innovation variance. We classify the cross-sectional results on 143 explosive phases into relatively low, middle and high values. We test a number of common beliefs in the literature and provide new insights into typical empirical properties of explosive prices and their collapse. Our results indicate significant discrepancies with typical settings in the literature. Empirical explosiveness is much milder and collapse phases are in most cases smooth rather than abrupt. Moreover, prices do not revert back to the initial value, but stay significantly above. The simplified view that prices are strongly exploding with a full collapse in short time is not supported by our results. Duration dependence modelling reveals that the length of the explosive phase is positively affected by economic growth, while the collapse duration is only driven by the length of the preceeding explosive phase in a positive way. Finally, we offer empirically relevant parametrizations for data generating processes and study the consequences for the empirical performance of popular bubble detection and date-stamping procedures.

Keywords: Explosive prices, stylized facts, duration, market recovery, volatility shifts

JEL classification: C15 (Statistical Simulation Methods), C22 (Time-Series Models), C41 (Duration Analysis), C58 (Financial Econometrics), G01 (Financial Crisis)

*__This is preliminary work in progress.__ The authors would like to thank Karim Abadir, Vasyl Golosnoy, Yuze Liu and participants of the Statistical Week (2021, Kiel), 15th International Conference Computational and Financial Econometrics (2021, London), 6th UA RuhrMetrics Seminar (2022, Duisburg) and RCEA Conference on Recent Developments in Economics, Econometrics and Finance (2022, The Rimini Centre for Economic Analysis) for helpful comments and discussions. Kruse-Becher gratefully acknowledges financial support from CREATES funded by the Danish National Research Foundation (DNRF78).

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1 Introduction

Due to the extensive number of financial exuberances and crises that took place during the last 30 years (e.g., dot-com bubble, sub-prime mortgage crisis, European debt crisis) and their serious consequences on whole economies and societies as well as their contagion effects, there is an ever-growing interest in how to deal with such extreme situations. Many authors investigate bubbles and exuberance periods in financial time series. Phillips, Wu, and Yu (2011) have developed a popular procedure to date stamp periods of financial exuberance. In order to be able to capture multiple explosive periods in a single financial time series, they offer an extended approach which is known as the Generalized Supremum Augmented Dickey Fuller (GSADF) Test in 2015 (Phillips, Shi, and Yu, 2015a; Phillips, Shi, and Yu, 2015b).

The goal of our research is to investigate the behaviour of daily real log-prices during the two phases of financial exuberance, namely the time from start to peak of the exuberance period (explosive period) and from the first day after the peak until the end of exuberance (mean-reverting period). While stylized facts[1] of financial time series in general are well-investigated (Pagan, 1996; Cont, 2001), there is not much known about the behaviour in different states of a financial time series. Therefore, we start by examining the empirical features of explosive phases and their corresponding mean-reverting market correction phase from a meta-analytical viewpoint.

The following general stylized facts have been demonstrated in the literature: log-returns of many financial time series do not show autocorrelation but instead autocorrelation is present in squared and absolute returns (Pagan, 1996; Cont, 2001). Furthermore, many time series show power-law distribution similarities but when increasing the latency of the data, e.g., from daily to yearly, the distribution becomes more and more like a normal distribution (Mandelbrot, 1963; Fama, 1965; Mandelbrot, 1967). Another finding is a gain/loss asymmetry as well as a phenomenon called "volatility clustering" (Engle, 1982; Bollerslev, 1986). In such a case, periods of low volatility tend to be followed by low volatility and periods of high volatility by high volatility. Another important finding are heavy tails, so that the emergence of extreme events is much more likely than in the case of a normal distribution (Mandelbrot, 1963; Fama, 1965; Mandelbrot, 1967). Besides these findings, also a leverage effect has been identified. It states that many volatility measures are negatively correlated with returns (Glosten, Jagannathan, and Runkle, 1993; Zakoian, 1994).

[1] Characteristics that are shared by many different kinds of financial time series are called "stylized facts" (Cont, 2001).

The major goal of our research is to close the gap between the literature on explosive price periods, bubble tests (and related econometric (monitoring) procedures) and stylized facts and subsequently, to develop new stylized facts. We are doing this by analyzing the identified exuberance periods returns concerning their distributional and dynamic properties for both the explosive and reverse period. Additionally, we analyze some general financial exuberance characteristics. Therefore, we look into how much value is typically gained during an explosive period and how much is lost in the reverse. Besides, we also investigate the duration of an exuberance period and the ratio of the duration of the explosive period compared to the reverse. Furthermore, we provide specific values for the autoregressive parameter in dynamic time series models during the explosive and reverse phase. Typical perceptions on these quantities might differ significantly from empirical features. Our whole analysis is done - in contrast to most of the existing literature on financial exuberance - based on real daily data.

The structure is as follows: Chapter two describes the used data set and in chapter three, the testing and identification procedure for financial exuberance periods is explained. In the upcoming fourth chapter, the identified price exuberance periods are described and additionally, we provide basic characteristics of financial exuberance periods. The fifth chapter investigates the stylized facts of these periods. We provide empirical evidence on 143 identified explosive phases and their characteristics. This section also includes Monte Carlo simulation results for the power of popular unit root tests against explosive alternatives based on empirically relevant specifications of the DGPs. Conclusions are drawn in Section 6.

2 Literature Review

Due to the severe systematic and societal consequences of financial turmoil and crisis (e.g., tulip mania, south sea bubble, great depression, dotcom bubble, global financial crisis), there is a rich history of research which deals with such situations. Nevertheless, the starting point is quite recently in 1978. Before, there was a common believe that financial crisis cannot be modelled mathematically. Kindleberger (1978)[2] set the basis by describing the theoretical aspects and consequences of manias, panics and crashes. Amongst his work, there have been published other famous books about the general structure of financial crisis, most prominently "This time is different: Eight centuries of

[2]Today, his book is available as the 7th edition. His work has been hold to life by Robert Z. Aliber after Kindleberger past away in 2003. So, for the 5th to 7th edition, Aliber has been responsible.

financial folly" by Reinhart and Rogoff (2009). Next to this more qualitative books, there is a great strand of literature about econometric and time series based bubble detection procedures. The first bubble test has been proposed by Flood and Garber (1980). Based on this research, Blanchard (1979)[3] showed that speculative bubbles do not collide with the rationality assumption. Then there is the category of variance bounds tests which has been proposed by Shiller (1981) and LeRoy and Porter (1981). Initially, they have not been developed for bubble detection but they are used by many authors for this purpose. Then in 1987, West introduced a two-step procedure (West, 1987; West, 1988). During the same time, Diba and Grossman (1987), Diba and Grossman (1988a), and Diba and Grossman (1988b) applied standard stationarity- and cointegration based tests for bubbles. Their approach has been famously criticised by Evans (1991) who especially pointed out that the above tests have poor power issues in detecting periodically collapsing bubbles. A detailed overview of econometric exuberance detection procedures up to the beginning of the 21st century can be found in Gürkaynak (2008). Another more recent category of bubble detection procedures is based on fractional integration tests, see e.g., Cunado, Gil-Alana, and Gracia (2005) and Frömmel and Kruse (2012).

Beginning with Phillips, Wu, and Yu (2011), the area of recursive unit root testing for bubbles gained popularity. They propose a right-tailed unit root test (supremum augmented Dickey-Fuller (SADF) test) which is not only able to detect exuberance periods but is also able to estimate the start and endpoint of a bubble. In the meantime, Homm and Breitung (2012) tested the power of different statistical procedures which have not been applied to exuberance detection so far and benchmarked them against the SADF test. They were able to show that two tests, namely one Chow-type Dickey Fuller test and a modified test of Busetti and Taylor (2004) show higher power than the SADF test. In 2015 then, the SADF test has been enlarged by Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) to solve the power issues if there are multiple periodically collapsing bubbles within a time series, which is typically the case. So, they propose a generalization of the SADF test, namely the GSADF test for explosiveness testing and the backward SADF (BSADF) test for date stamping. Both approaches are nowadays considered as the market standard and have been applied to numerous markets, e.g., Anundsen, Gerdrup, Hansen, and Kragh-Sorensen (2016) for housing, Corbet, B. Lucey, and Yarovaya (2018) for cryptocurrencies, Brunnermeier, Rother, and Schnabel (2020) for stock markets and systematic risk and Contessi, De Pace, and Guidolin (2020) for fixed

[3]Interestingly, nevertheless Blanchard based his analysis on Flood and Garber (1980), Blanchard's article has been published earlier.

income markets.

Besides the mentioned procedures, there is also the BIC-date stamping idea of Harvey, Leybourne, and Sollis (2017) and Harvey, Leybourne, and Whitehouse (2020). Phillips and Shi (2018) address the empirically highly relevant issue of smooth collapses in the context of bubble detection and date-stamping. In a similar vein, Monschang and Wilfling (2021) further investigate the empirical performance of such popular procedures by Monte Carlo simulations. Their work is closely related to our analysis. Müller and Elliott (2003), Harvey and Leybourne (2014) and Whitehouse (2019) analyse the impact of (non-)negligible initial conditions on unit root tests against stationary and explosive alternatives. The issue of time-varying volatility (i.e. the unconditional variance of innovations), is treated in Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne, and Zu (2019). The wild bootstrap approach has emerged as the standard procedure to conduct robust inference. It is important to mention that this list is by no means complete due to the rich amount of literature published about financial crisis.

3 Data

During the analysis, a diverse range of financial time series is analysed. This include equity market indices, precious metals, oil, cryptocurrencies and real estate indices. The advantage of this high diversity of financial time series is to search for stylized facts and not just for some asset specific findings.

All time series are downloaded as price indices from REFINITIV datastream (formerly known as Thomson Reuters datastream). In each case, we use daily observations which start at 2nd January 1970[4] or if data are not available from this point in time, the longest available history is used. Equity indices are chosen in such a way that not only the most important indices of the world are considered but also emerging and frontier markets as well as indices from countries which are spread all around the world. This is done to avoid a bias towards developed countries and to the biggest financial markets, especially towards the United States. Therefore, we use a total of 24 indices from Europe (AEX, CAC 40, DAX 30, FTSE 100, OMXH and SMI), America (Mexico IPC, NASDAQ, S&P 500 and S&P TSX Composite), Asia (Hang Seng, IDX Composite, KOSPI, NIFTY 500, NIKKEI 225, Shanghai SE A Share, Straits Times Index L and TOPIX), Africa (FTSE South

[4]Start is not on the 1st January 1970 because in most countries, stock exchanges were closed on this day.

Africa, HRMS, MASI and TUNINDEX) and the two intercontinental countries Israel (Israel TA 125) and Russia (MOEX).[5]

Table 1: stock market indices applied in the analysis

index	**datastream**	**currency**	**country**	**start date**	**end date**
AEX	AMSTEOE	EUR	The Netherlands	03.01.1983	30.04.2021
CAC 40	FRCAC40	EUR	France	01.01.1990	30.04.2021
DAX 30	DAXINDX	EUR	Germany	02.01.1970	30.04.2021
FTSE 100	FTSE100	GBP	United Kingdom	01.01.1988	30.04.2021
FTSE South Africa	JSEOVER	ZAR	South Africa	30.06.1995	30.04.2021
Hang Seng	HNGKNGI	HKD	Hong Kong	01.10.1980	30.04.2021
HRMS	EGHFINC	EGP	Egypt	02.01.1995	30.04.2021
IDX Composite	JAKCOMP	IDR	Indonesia	01.01.1996	30.04.2021
Israel TA 125	ISTA100	ILS	Israel	23.04.1987	30.04.2021
KOSPI	KORCOMP	KRW	Republic of Korea	31.12.1974	30.04.2021
MASI	MASIIDX	MAD	Morocco	01.01.2007	30.04.2021
Mexico IPC	MXIPC35	MXN	Mexico	04.01.1988	30.04.2021
MOEX	RSMICEX	RUB	Russian Federation	22.09.1997	30.04.2021
NASDAQ	NASCOMP	USD	United States	05.02.1971	30.04.2021
NIFTY 500	ICRI500	INR	India	03.01.2011	30.04.2021
NIKKEI 225	JAPDOWA	JPY	Japan	02.01.1970	30.04.2021
OMXH	HEXINDX	EUR	Finland	02.01.1987	30.04.2021
S&P 500	S&PCOMP	USD	United States	02.01.1970	30.04.2021
S&P TSX Composite	TTOCOMP	CAD	Canada	02.01.1970	30.04.2021
Shanghai SE A Share	CHSASHR	CNY	China	02.01.1992	30.04.2021
SMI	SWISSMI	CHF	Switzerland	30.06.1988	30.04.2021
Straits Times Index L	SNGPORI	SGD	Singapore	31.08.1999	30.04.2021
TOPIX	TOKYOSE	JPY	Japan	02.01.1970	30.04.2021
TUNINDEX	TUTUNIN	TND	Tunisia	31.12.1997	30.04.2021

This table provides an overview of all applied stock market indices in the analysis. They are sorted alphabetically based on their first letter. The first column provides the name of each index and the second the datastream symbol that has been used to obtain the data. In the third column, the ISO 4217 currency codes for the indices are provided. The next column illustrates the country which is covered by the specific index and the last two columns provide the starting and end date of the index which is used in the analysis section.

In addition to stock market indices, we also consider the two precious metals gold and silver which are often used and assumed as safe heaven assets due to their low correlation to stock and bond market indices and they are also seen many times as assets which can protect against inflation (Hillier, Draper, and Faff, 2006; Bauer and B. M. Lucey, 2010; Bauer and McDermott, 2010; Bampinas and Panagiotidis, 2015). Besides, to get a broader picture, also the two oil indices Crude Oil WTI[6]

[5] An overview of all applied stock market indices is available in Table 1.

[6] To be able to apply the logarithm, the negative value of USD 37.63 at 20th April 2020 is replaced by USD 0.0001. This negative value occurred due to the lag of storage of oil producers due to a lag of demand for WTI oil and was a unique event never happened before (Corbet, Goodell, and Günay, 2020).

and Europe Brent are investigated.[7] On top of this, Bitcoin is considered as a representative of the new asset class cryptocurrencies. Especially against the background of the global financial crisis of 2007/09, also the DJ US Real Estate Index is investigated.[8]

Table 2: indices of other assets than stocks applied in the analysis

index	**datastream**	**currency**	**asset category**	**start date**	**end date**
Bitcoin	BTCTOU$	USD	cryptocurrency	18.08.2011	30.04.2021
Crude Oil WTI	CRUDOIL	USD	oil	10.01.1983	30.04.2021
DJ US Real Estate	DJUSRE$	USD	real estate	02.01.1992	30.04.2021
Europe Brent	EIAEBRT	USD	oil	20.05.1987	30.04.2021
Gold	GOLDBLN	USD	precious metal	01.01.1980	30.04.2021
Silver	SILVERH	USD	precious metal	01.01.1980	30.04.2021

This table provides an overview of all applied assets in the analysis which are no equity indices. They are sorted alphabetically based on their first letter. The first column provides the name of each asset/index and the second the datastream symbol that has been used to obtain the data. In the third column, the ISO 4217 currency codes for the indices are provided. The next column states the asset category which is covered by the specific index and the last two columns illustrate the starting and end date of the index which is used in the analysis section. Due to the reliability of data, Gold and Silver are only considered from the first January 1980 and not prior.

All of these time series price data (P_t) are used twofold. On the one hand, log nominal price data are used because market sentiment is most often build on such prices (Baker and Wurgler, 2007). On the other hand, we also perform our analysis based on real log prices, so that price increases simply driven by inflation are cancelled out. This is done by dividing the assets'/indices' price by the consumer price index (CPI) of the corresponding country/region:[9]

$$p_{t_{real}} = \ln\left(\frac{P_t}{CPI_t}\right). \tag{1}$$

These data are later applied in the financial exuberance detection and data stamping procedures.

[7] Because gold, silver and oil are considered as homogenous assets, we decide to focus on a few hand selected time series of these assets because the use of more time series would not result in an additional information gain.

[8] Table 2 provides an overview of the used assets other than stock market indices.

[9] Due to the fact that CPIs are only available on a monthly basis, the monthly CPI corresponding to each index's/asset's currency is used as a proxy for the daily price level. Some time series are shorten because of the later availability of CPI values. Furthermore, all price data are scaled by 100 to avoid situations where the logarithm of the real price is negative. This is done to do not have problems with situations where the start of exuberance is with a negative price and its peak is positive because in such situations, one cannot reasonably calculate growth rates.

4 Econometric methods

In the literature, there are many different tests for identifying periods of financial exuberance. The most often applied models are based on the idea of rational bubbles. In this chapter, the well-known Generalized Supremum Augmented Dickey Fuller (GSADF) technique of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) and their date stamping procedure (Backward Supremum Augmented Dickey Fuller (BSADF) Test) are described.

4.1 Testing for multiple explosive prices

The idea is to apply a recursive regression procedure based on ADF tests because it has been demonstrated in the literature that standard unit root and cointegration tests are not able to detect multiple collapsing bubbles within the same time series (Flood and Garber, 1980; Flood and Hodrick, 1986; Evans, 1991). The right-tailed unit root test is constructed based on the following ADF-regression which is simply estimated by ordinary least squares (OLS):

$$\Delta y_t = \mu_{r_1,r_2} + \rho_{r_1,r_2} y_{t-1} + \epsilon_t. \tag{2}$$

The first difference of log real prices Δy_t (which are log returns) is regressed on the sum of a slope μ_{r_1,r_2}, a ρ_{r_1,r_2} weighted first order lag of the log real price y_{t-1}, and the error term ϵ_t. This ADF regression is estimated multiple times by using different subsets of the sample data. The null hypothesis of a unit root is then tested against the alternative hypothesis of a (mildly) explosive process:[10]

$$H_0 : \rho_{r_1,r_2} = 0 \text{ (unit root)},$$
$$H_1 : \rho_{r_1,r_2} > 0 \text{ (mildly explosive behaviour)}.$$

The normalized subset's start and end values r_1 and r_2 are both defined in such a way that they are allowed to grow. The minimum value of r_1 is 0. So, r_1 starts with the first observation of the applied data set. Its maximum is set to the difference between the value of r_2 and the minimum window size r_0. In contrast, r_2 runs from r_0 to the latest (most new) observation (1) in the data

[10] Equivalently, often the following version of the ADF regression is used: $y_t = \mu_{r_1,r_2} + \rho_c y_{t-1} + \epsilon_t$. In this setting $\rho_c = \rho_{r_1,r_2} + 1$ and consequently, the null hypothesis $\rho_c = 1$ is tested against the alternative of $\rho_c > 1$ (Contessi, De Pace, and Guidolin, 2020).

set:

$$r_1 \in [0, r_2 - r_0],$$
$$r_2 \in [r_0, 1].$$

The minimum window size r_0 is defined as $0.01 + 1.8/\sqrt{T}$, where T illustrates the number of observations. The size of the subsample is increased by one observation until the limit is reached - the last observation of the used data set.

The GSADF test is then stated as:

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0,1] \\ r_1 \in [0, r_2 - r_0]}} \{ADF_{r_1}^{r_2}\}. \tag{3}$$

To identify if financial exuberances are in the sample, the calculated GSADF statistic is compared to its critical value of the distribution under the null hypothesis. This critical value is determined by a bootstrapping procedure proposed by Phillips and Shi (2020) which accounts for potential heteroskedasticity issues. This procedure consists of five steps. First, the ADF regression model is estimated under the hypothesis that ρ is 0 based on the total available data set. In the second step, a bootstrap sample is constructed and after it, the PSY test statistic series and based on it, the maximum value is calculated. Next, these two steps are repeated n times (in this research, 300 times). In the last step, the 95% quantile of the series of maximum values is calculated and this is the critical value of the test. If there are no exuberance periods found in the time series, the time series is considered to do not show financial exuberant prices at any time and therefore drops out of the following date stamping and analysis.

4.2 Identification of explosive periods

The previously described approach is able to detect the existence of exuberant prices but it does not allow for date stamping. To locate the begin and end of exuberance, the so-called backward SADF (BSADF) test of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) is applied.[11]

[11] We apply these procedures due their vast popularity in applied work. Thereby, we enable comparison to existing results in the empirical literature. However, it might be interesting to compare the outcomes to those obtained from CUSUM-based and BIC procedures in future work. The latter ones are typically performing somewhat better, see Homm and Breitung (2012), Breitung and Kruse (2013), Harvey, Leybourne, and Sollis (2017), Whitehouse (2019) and Harvey, Leybourne, and Whitehouse (2020).

It performs ADF tests based on a backward expanding sample which has a fixed endpoint r_2 but varying starting points r_1:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \{ADF_{r_1}^{r_2}\}. \tag{4}$$

The initiation date $\hat{r}_e$ is the first time, where the test statistic exceeds its critical value $(scv_{r2}(\beta_T))$ and the termination date $\hat{r}_f$ is the date at which the test statistic first lies below its critical value:

$$\hat{r_e} = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}(\beta_T)\}, \tag{5}$$

$$\hat{r_f} = \inf_{r_2 \in [\hat{r_e}, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}(\beta_T)\}. \tag{6}$$

To make the identification technique of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) more robust, to account for the fact that no econometric model is perfect and to produce better results, we make two adjustments. First, we only consider periods of financial exuberance which have a duration of at least 66 business/trading days.[12] Shorter periods are excluded to reduce the influence of noise and to account for the fact that no exuberance detection model is perfectly designed. On top of this, small duration periods are hardly considered as being a financial exuberance period.[13] Second, we distinguish between positive and negative exuberance periods. While positive exuberances consists of an explosive increase which is followed by a reverse, negative exuberances start with a decrease in value and an upcoming increase.[14] Our main focus is on positive exuberance periods because they are much more common in practice. But we do not neglect negative exuberances like traditional research of Blanchard and Watson (1982) and Diba and Grossman (1988a) because recent research has shown that negative bubbles are possible and can be observed in financial markets (Fry and Cheah, 2016; Goetzmann and Kim, 2018; Phillips and Shi, 2018; Acharya and Naqvi, 2019). Instead we additionally provide all analysis results of

[12]The duration is set based on Phillips and Shi (2018). They use monthly data and require three subsequent observations that are detected by the BSADF procedure to be considered as a financial exuberance period. Because we are using daily data, we scale the number of required observations to 66 trading days which is the approximate number within a three month period.

[13]All financial exuberance periods with a duration of at least 66 trading days are used to analyse the basic characteristics of exuberance periods like duration, starting value, peak value, end value etc. But in the upcoming analysis of stylized facts, the minimum duration of both financial exuberance parts (explosive and reverse period) is slightly adapted because otherwise, the applied models have not enough observation data and could lead to false/biased results. This is a trade-off which automatically emerges while combining the areas of financial exuberance and stylized facts.

[14]For clarity, in the later analysis, we use the terminology shock period for the decrease in value and recovery period for the later value increase in a negative financial exuberance period. Additionally, the minimum value is called bottom value.

negative exuberances in an excursus.

5 Identified periods of crisis

In this section we intensively discuss the identified exuberance periods based on log real prices for both positive and negative exuberance periods. To test for sensitivity/robustness and to provide further insides, we also state the exuberance detection results based on log nominal prices in the appendix.

Based on the BSADF test, we have identified positive exuberances in 29 out of all 30 investigated time series. Only WTI Oil does not show such behaviour. In these 29 time series, we detect 146 positive exuberances.[15] In contrast, we identify negative exuberance periods in only 23 out of all 30 assets/indices and the total amount of negative exuberances is 63. Before we start to analyse the stylized facts of these periods, we provide some basic characteristics concerning the financial exuberance phases themself.[16]

First, we provide basic characteristics of positive financial exuberance periods. Duration is defined as the number of trading days a financial exuberance period consists of. The mean value in our sample is 307.06 days with a median value of 150. The shortest exuberances are only 66 days and they have been identified in the Brent Oil time series for the period running from 2008-05-02 to 2008-08-01 and in the DAX30 time series running from 1989-07-14 to 1989-10-13. In contrast, the longest identified exuberance periods were observable in TOPIX (1983-10-27 to 1991-11-29, 2112 trading days) and TUNINDEX (2005-04-05 to 2013-05-27, 2125 trading days). The mean duration is mainly driven by the duration of the explosive period which is on average 2.52 (219.90/87.16) times longer than that of the reverse period. Looking on the average duration ratio rather then dividing the mean explosive duration by the mean reverse duration, the ratio is even bigger (4.90). Similar but smaller results are obtained by using the median value instead. In this case, the explosive period is more than three times as high (3.08, respectively 3.10) than the reverse period. Another finding is the much higher increase during the explosive period compared to the decline in the reverse. The increase is approximately 41% (based on mean) or 22% (based on median) while the decrease is 19% (mean) or 11% (median) (Table 3).

[15] Three exuberance periods are dropped out because they are not fully burst at the end of our observation sample. These exuberances are in BITCOIN, NASDAQ and S&P 500. Using them would bias our results because we would analyse exuberance periods which are not over. So, we have in total 143 positive exuberance periods.

[16] All identified exuberance periods, both positive and negative ones, are shown with some details in the appendix.

Table 3: Basic characteristics of positive financial exuberance periods based on log real prices

	mean	median	sd	min	max	5%Q	95%Q
duration	307.06	150	407.46	66	2125	69.10	1338.50
explosive duration	219.90	114	299.49	16	1603	41.00	948.50
reverse duration	87.16	37	125.89	2	692	8.00	394.90
duration ratio	4.90	3.10	5.82	0.21	47.00	0.51	15.11
increase explosive	0.41	0.22	0.56	0.05	4.42	0.07	1.30
decrease reverse	-0.19	-0.11	0.22	-1.52	-0.02	-0.58	-0.03

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

These overview clearly illustrates that the often made assumption of a 1-period-crash [17] is in general not justified for the well-known financial exuberance periods. Additionally, after the exuberance is over, the real asset price does not reach the level which it had before the exuberance started. The new value is most often much higher compared to the initial value. Keeping this in mind, we will later construct new data generating processes (DGP) which are much more realistic and backed by empirical results compared to the DGP of Evans (1991). Especially, we account for the diversity of financial exuberance periods, so that we develop more than one DGP. In line with this, we additionally provide all exuberance characteristics and analysis results in a more detailed way. Therefore, to make our analysis more expressive and detailed, we additionally provide all analysis results sorted based on the two dimensions duration and duration ratio (Table 4). The majority of identified financial exuberance periods (102) has a duration of less than a year. 27 exuberances have a duration of one year or are smaller than two years and 16 exuberances last two or more years. The second applied dimension is the duration ratio. Approximately half of the exuberance periods of each of the three duration categories has a duration ratio of under 3 and the other half of equal or bigger than 3. The detailed results are provided in the appendix and in the part in which we develop new DGPs we will refer to them.

Table 4: Analysed subsamples based on log real prices

	duration ratio < 3	duration ratio ≥ 3	$\sum$
duration < 1 year	48	52	102
duration $\in [1, 2)$ years	15	12	27
duration > 2 years	8	8	16

[17] Evans (1991) bubble DGP which is based on monthly data, assumes that there is a "hard" crash within one month.

In contrast to the existing literature, we also consider negative financial exuberance periods (Table 5). The mean value of negative financial exuberance periods is 172.52 and thus, it is significantly shorter compared to positive exuberances. Comparing the median of both types of financial exuberance, the difference becomes slighter (129 vs. 150 days). The shortest period with a duration of 66 days has been found in the WTI Oil time series between 2015-07-06 and 2015-10-05. In contrast, the longest observable negative exuberance has been identified in OMXH (1989-09-21 to 1993-03-01, 898 trading days). The average duration is mainly driven by the duration of the shock period (112.29 based on mean, 71 based on median) which is underlined by an average duration ratio of 3.53 and a median duration ratio of 1.72. Therefore, the difference in duration between both parts of a negative exuberance is less compared the difference in positive exuberances. Having a closer look at the decrease and increase during the shock and recovery it becomes obvious that the average shock (-0.40 or -0.37) is much larger than the average recovery (0.20 or 0.19). So, in contrast to positive exuberances, the end value is on average lower than the starting value while in positive exuberances it is the opposite. Keeping the made findings in mind, it is essential to also consider negative financial exuberances because they are not that uncommon in reality like assumed by some authors and additionally, the exuberance characteristics are different compared to the "normal" financial exuberance periods.

Table 5: Basic characteristics of negative financial exuberance periods based on log real prices

	mean	median	sd	min	max	5%Q	95%Q
duration	172.52	129	145.75	66.00	898	70	432.10
shock duration	112.29	71	127.76	12	773	26.40	275.30
recov. duration	60.24	41	49.70	6	265	8.70	144.90
duration ratio	3.53	1.72	5.58	0.10	36.22	0.36	10.51
decrease shock	-0.40	-0.37	0.27	-1.30	-0.06	-0.92	-0.09
increase recov.	0.20	0.19	0.13	0.03	0.58	0.04	0.41

This table provides an overview of the basic characteristics of the identified negative financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digit while median, minimum value (min) and maximum value (max) are stated in integers for duration, shock duration and recovery (recov.) duration.

Another important information when dealing with often applied AR(1) models is the value of the AR parameter ρ.[18] Therefore, we provide evidence that in the explosive period, ρ is, like expected, slightly greater than 1. In all investigated exuberance periods, it is always greater than 1. The lowest observed value is 1.00008 while the highest one is 1.01627. Its average value is 1.00049 based

[18] To avoid estimation issues due to very small periods of explosiveness or reverse, we eliminate all periods with less than 10 trading days => not done so far!

on mean and 1.00023 based on median. In contrast, in each reverse period, ρ lies between 0.99771 and 0.99996. Thus, it is close to unity but slightly smaller than 1 and has an average value of 0.99953 (mean) or 0.99967 (median) (Table 6). Knowing these specific values is essential to be able to state an empirical justified data generating process. Therefore, in the appendix are more detailed values for our six subsamples.

Table 6: AR(1) model parameters of positive exuberances

	mean	median	sd	min	max	5%Q	95%Q
AR1 expl	1.00049	1.00023	0.00145	1.00008	1.01627	1.00010	1.00126
AR1 rev.	0.99953	0.99967	0.00044	0.99771	0.99996	0.99848	0.99993

This table provides an overview of the AR(1) model parameter for both the explosive and reverse period. Provided are the mean, median, standard deviation (sd), minimum value (min), maximum value (max), 5% (5%Q) and 95% quantile (95%Q). All values are rounded to five digits.

Similar results can be obtained for negative financial exuberance periods. The ρ levels of shock periods are in a comparable range of the reverse periods of positive exuberances and ρ of the recovery is similar to that of the explosive part (Table 7).

Table 7: AR(1) model parameters of negative exuberances

	mean	median	sd	min	max
AR1 shock	0.9990	0.9994	0.0011	0.9937	0.9999
AR1 recov.	1.0009	1.0005	0.0011	1.0000	1.0058

This table provides an overview of the AR(1) model parameter for both the shock and recovery period. Provided are the mean, median, standard deviation (sd), minimum value (min) and maximum value (max). All values are rounded to four digits.

To make our analysis even stronger and more sound, we not only investigate the explosive and reverse period but also have a closer look at the time before and after the exuberance period which serve as a benchmark in our analysis. The benchmarks are chosen based on the number of business days in the exuberance period. The period before and after are chosen so that the number of business days is equal to those in the exuberance period. If this is not possible because the duration between two separate exuberance periods is not long enough, the minimum requirement is to use a sample which is at least 66 trading days long. If this is not possible, we consider the benchmark is not available. It is important to note that we do not allow overlapped post exuberance periods of the finished exuberance period with the pre period of the next exuberance period. The applied benchmark periods are stated in Table XX. For our later developed data generating processes (DGPs) we group our data into three different categories (low, mid, high):

Table 8: Plain vanilla DGP parameter values

		group 1	group 2	group 3
1	p_0	4.51	6.76	8.94
2	T_e;T_r	(81;28)	(226;99)	(815;350)
3	ν	4	5	6
4	σ_e^2;σ_r^2	(0.000057;0.000096)	(0.000156;0.000329)	(0.001008;0.001278)
5	ρ_e	1.000163	1.000352	1.002027
6	ρ_r	0.998638	0.999480	0.999804

This table provides an overview of the eight variables/parameters set for the DGP. While the starting price p_0 is rounded to two digits, both variances and AR coefficients are round to six digits due to their smallness. Duration and variances for both periods are stated in tuples because empirically mixing these parameters from different groups is not observable.

Logarithmic durations (explosive expansion and mean-reverting collapse phases) are investigated in terms of their distributional characteristics. We find that the normality hypothesis for both durations cannot be rejected at any conventional significance level. The p-values for the Jarque and Bera (1980) test are equal to 0.51 and 0.26 for explosive and mean-reverting durations, respectively. This result motivates the use of a log-normal distribution for the survival regressions involving the durations in level.

First, we test the common belief that prices collapse within a single period. As the minimal reverse duration is equal to two days, a test based on daily periods can only reject such a null hypothesis. However, most econometric procedures are simulated on a monthly basis, such that we might instead test the one-sided null hypothesis that the collapse duration is smaller or equal to 22 trading days (which corresponds to a single period on a monthly frequency). Based on normality of log-durations, we obtain a t-statistic of 7.07.[19] Hence, the null hypothesis is clearly rejected in favor of the alternative that durations exceed 22 days on average.

Second, we test the hypothesis of a full collapse (irrespective of its duration). A full collapse is characterized by a complete reversion of the price to the initial pre-explosive value. In other words, the market correction brings the price back to its value before the exuberance period started. This might take place within a single period (sudden collapse) or within a prolonged phase (disturbing collapse). The former possibility is already rejected based on our previous findings. What remains is the possibility of a disturbing or smooth collapse. One-sided testing of the null hypothesis of a full collapse leads to a heteroskedasiticity-robust t-statistic of 4.46. Taking the evidence together, the form a smooth collapse — as innovated in Phillips and Shi (2018) — is clearly supported on

[19] Throughout the analysis, we entertain Huber-Eicker-White robust standards errors against cross-sectional heteroskedasticity.

average.

The following return scatterplot (similar to Etienne, Irwin and Garcia, 2014) shows a clear linear relationship between the cumulated returns for the phase (i) from peak to burst (y-axis) and (ii) start to peak (x-axis) ($R^2 = 0.822$). The returns from start to peak are significantly larger in magnitude as the reversions from peak to burst. An interesting outlier is one of Bitcoin cases where the log-return in the explosive phase exceeds 400%, while its reversion is less than 100%.

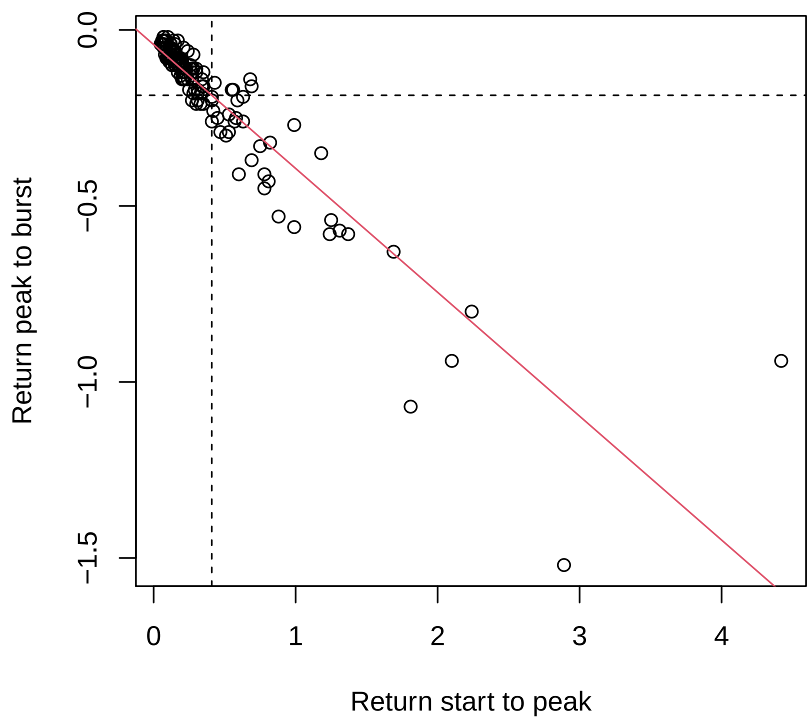

Figure 1: Returns during explosive and reverse period

Turning to the duration dependence modelling (see e.g. McQueen and Thorley, 1994 and Lunde and Timmermann, 2004), we investigate the role of annual US real interest and growth rates on the durations of explosive and collapse phases. The macroeconomic data is obtained from the FRED and matched to the beginning of the explosive phase, similar to Kennan (1985) for the modelling of strike durations. Based on our previous investigations, we apply a parametric log-normal survival model. Contrary to Lunde and Timmermann (2004), we do not find evidence for the effect of interest rates $t = 1.33$. However, it must be noted that the authors consider bull and bear markets which are different from the phases we investigate here, albeit clearly related in concept. We find real GDP growth to have a positive and significant impact on the length of explosive regime durations

($t = 2.14$). Moreover, collapse durations are not driven by economic measures, but strongly depend on the length of the preceding explosive phase ($t = 9.60$). In short, long explosive phases do not collapse quickly, but rather take their time ($R^2 = 0.302$).

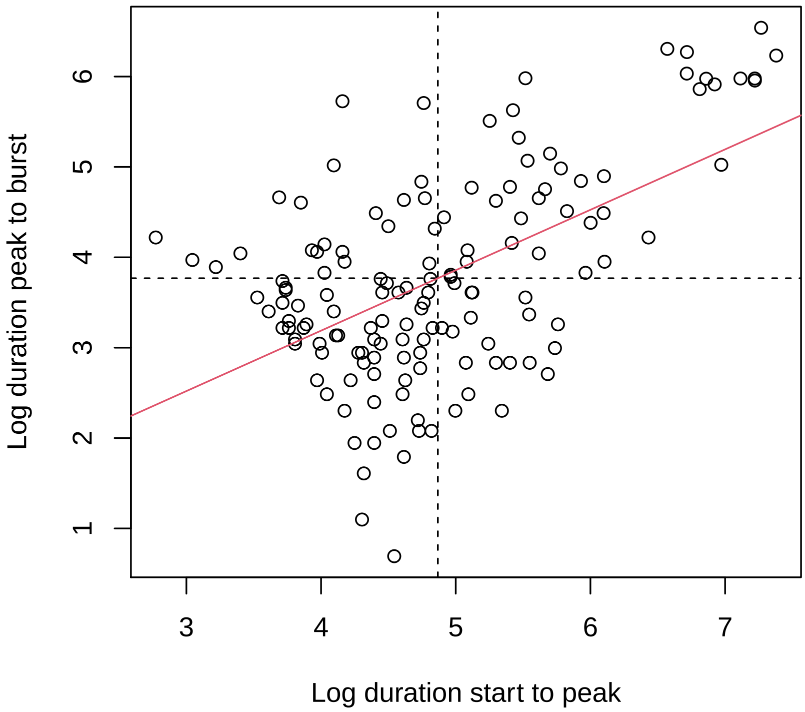

Figure 2: Log durations during explosive and reverse period

Furthermore, we look at the estimated autoregressive parameters during expansions and market recovery phases. For the comparison of daily explosive autoregressive parameters (ρ_d) to monthly counterparts (ρ_m) — as typically considered in the related literature — we apply the simple conversion $\rho_m = \rho_d^{22}$. Our results suggest monthly explosive autoregressive parameters of 1.004 (low); 1.008 (mid) and 1.046 (high). They clearly indicate that typical autoregressive parameters used in Monte Carlo simulations are far too high and lead to too optimistic results as the econometric procedures clearly benefit from large autoregressive roots exceeding unity. Testing (one-sided) the null hypothesis that the average autoregressive parameter we find empirically equals 1.02 on a monthly basis (usually the minimal value for explosive roots, see e.g. Homm and Breitung, 2012 and many others) leads to $t = 3.37$. Hence, the null hypothesis is clearly rejected in favour of smaller autoregressive parameters. In our Monte Carlo simulations, we investigate the impact of these findings on the power of the popular ADF-type tests againts explosiveness.

Finally, we test the null hypothesis of no structural break in the innovation variance during the explosive and the collapse phase. The resulting t-statistic equals 7.61 and thus confirms the existence of a structural change in the unconditional variance. Moreover, we are able to quantify typical break sizes. These are 1.27 (low); 1.69 (mid) and 2.11 (high). Compared to the existing literature, these break sizes are relatively small.

6 Stylized facts

There is much known about general stylized facts of financial time series (Pagan, 1996; Cont, 2001) but until now, to the best of our knowledge, there are no widely established stylized facts concerning the behaviour of log-returns in financial exuberance periods. Furthermore, this is also true for the filtered residuals of an AR(1) model in this context. Therefore, we investigate the autocorrelation, distribution, volatility and leverage effect characteristics. Additionally we also do duration modelling. These investigations are first done on the complete financial exuberance periods and in the second step, we further divide each exuberance into its explosive and reverse part to investigate if there are major differences between both parts of an exuberance. After that, we compare our results to periods of non-financial exuberance and the stylized facts of the whole financial time series without considering structural breaks to illustrate the differences and to make our results more reliable.

6.1 Distribution

Since Mandelbrot (1963), Fama (1965), and Mandelbrot (1967) there is evidence that most financial time series exhibit heavy tails and thus, the log returns do not follow an often assumed normal distribution. Instead it has been shown that the process looks more like a Pareto or power law distribution. In the following, we investigate if this finding can be confirmed for exuberance periods and both its explosive and reverse part. The first step in our analysis is therefore to create QQ-plots to visually analyse this task. Next we estimate the sample skewness and kurtosis, the Jarque-Bera test statistic[20] and finally, we determine the tail-index.

[20]The Jarque-Bera statistics are only provided for periods which have a duration of at least 264 trading days (one trading year) because in small samples, Jarque-Bera statistics show quite poor size and power characteristics. We are often confronted with small sample issues during our analysis. Therefore, significance tests are only provided for long enough exuberance, explosive and reverse periods (Cont, 2001). For all other periods, we concentrate on estimating the statistics itself but we avoid testing them. At first sight, this seems unsatisfactory but because our goal is to model more realistic data generating processes, the issue does not seem to big.

The QQ-plots clearly illustrate that the great majority of investigated exuberance periods as well as both subperiods strongly differ from a normal distribution. Due to the high number of QQ-plots (for the positive exuberances based on log real prices, these are alone 3×143 QQ-plots), they are not printed here and are available on request. Next, we calculate the skewness (s) and kurtosis (k) to determine if they differ from the Gaussian distribution (McNeil, Frey, and Embrechts, 2015):[21]

$$s = \frac{\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^3}{\left[\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^2\right]^{1.5}}, \tag{7}$$

$$k = \frac{\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^4}{\left[\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^2\right]^{2}}. \tag{8}$$

The skewness is found to be negative on average for all three periods which is a well-known stylized fact of exuberance periods (Cont, 2001). Based on both mean and median, the skewness is most negative for the complete exuberance (-0.45 and -0.46) compared to its subperiods (-0.23 and -0.23 for explosive period and -0.33 and -0.25 for reverse period). Comparing the skewness of the explosive and reverse period, the mean of the reverse period is more negative but based on median, there are hardly differences between both parts. Based on the standard deviation, minimum and maximum value as well as on the 5% and 95% quantile, it is obvious that there is a high fluctuation between the skewness of the investigated exuberances. This is not surprising due to the fact that many exuberances and crisis are driven by different causes like great, moral hazard, strong currency depreciations/appreciations, credit and housing markets turmoil etc. The kurtosis also differs for all three periods. The mean and median values are both higher for the complete exuberance (6.03 and 5.91) rather than for both subperiods (explosive: 5.91 and 3.99, reverse: 4.22 and 3.21). Comparing both subperiods, the kurtosis is higher during the explosive period. The following table provides an overview of these two moments (table 8):

Therefore, to get deeper insides into the characteristics and stylized facts of exuberance periods, we do regression and hazard rate analysis in which we detect connections between various aspects of an exuberance like duration ratio, skewness, kurtosis, standard deviation, autoregressive parameter etc.

In the last step, to further confirm if there are heavy tails, we apply the weighted Hill estimator

[21] Because normality tests like the Jarque-Bera test are developed asymptotically, we perform size and power analysis based different functions like student distributions, Frechet distributions etc. We conclude that Jarque Bera statistics suffer strongly in size and power for distributions of heavy tails with less than 200 observations. Therefore, to be consistent, we only perform such tests for exuberance periods which lasts at least for one trading year (264 observations).

Table 9: Skewness and kurtosis of exuberance, explosive and reverse periods for positive exuberances based on log real prices

		mean	median	sd	min	max	5%Q	95%Q
	exub	-0.45	-0.46	0.65	-2.38	2.58	-1.47	0.60
skew	expl	-0.23	-0.23	0.75	-3.07	2.93	-1.24	0.93
	rev	-0.33	-0.25	0.64	-3.31	0.98	-1.39	0.52
	exub	6.03	4.54	4.72	2.50	32.14	2.90	14.48
kurt	expl	5.91	3.99	7.78	2.21	66.54	2.54	15.16
	rev	4.22	3.21	2.82	1.46	18.88	2.05	10.33

of Huisman, Koedijk, Kool, and Palm (2001). This estimator fixes the small sample issue and the challenge of choosing the threshold value k which emerges in the original Hill estimator (Hill, 1975). The Hill estimator for the largest positive returns is defined as:

$$\xi(k) = \frac{1}{k}\sum_{i=1}^{k}[\ln(r_{T-i+1}) - \ln(r_{T-k})]. \tag{9}$$

It is applied to increasing sorted returns, such that $r_1 \leq r_2 \leq \cdots \leq r_T$. But because risk management is mainly concerned with the most negative returns and the Hill estimator is only able to handle positive values, all returns are multiplied by (-1) to obtain losses, so, e.g., a -5% return is a 5% loss. Now, Huisman, Koedijk, Kool, and Palm (2001) uses the Hill estimator definition but instead of only calculating the Hill estimator for one specific chosen k, the idea is to calculate the Hill estimator for the threshold values $k \in \{1, 2, \ldots, \kappa\}$ and then, the weighted Hill estimator is defined as:

$$\xi^m(\kappa) = \sum_{k=1}^{\kappa} w(k)\xi(k). \tag{10}$$

The weighted hill estimator is calculated based on weighted least squares (WLS) and it is the first element of the vector b_{WLS}:

$$b_{WLS} = (Z'W'WZ)^{-1}Z'W'W\xi^*. \tag{11}$$

Z is a $\kappa \times 2$ matrix with 1's in its first column and $k \in \{1, 2, ..., \kappa\}$ in its second column. W is a $\kappa \times \kappa$ weighting matrix with $\{\sqrt{1}, \sqrt{2}, \ldots, \sqrt{\kappa}\}$ on its diagonal and Os everywhere else. Additionally, ξ^* is a vector containing all Hill estimates up to the maximum threshold κ. The maximum threshold value is then chosen in line with Huisman, Koedijk, Kool, and Palm (2001) who suggest $\delta = T/2$. Estimating the parameters using OLS would result in two major issues, namely neglecting heteroscedasticity and correlation between the variables $\gamma(k)$. Therefore, a weighted least squares (WLS) approach is applied. Based on the weighted Hill estimator the tail index α is defined as:

$$\alpha = \frac{1}{\xi}. \tag{12}$$

Applying this procedure to our data, we obtain that on average and based on median as well, the tail index is about 4 to 6 what means that the underlying stochastic process has a finite fourth moment. Furthermore, with only a handful of exceptions, each exuberance period and its subperiods have a finite variance. Furthermore, we find support of the gain/loss asymmetry because the tail indexes for losses are smaller and thus, more extreme than the tail estimates for the largest returns (table 9). To summarize, most exuberance, explosive and reverse periods show signs of non-normality and

Table 10: Tail estimator of largest returns and losses

		mean	median	sd	min	max	5%Q	95%Q
	exub	6.44	5.48	4.25	2.50	25.07	2.91	13.26
positive tail	expl	6.14	5.61	2.92	2.19	18.06	2.82	10.44
	rev	5.94	4.50	4.05	2.47	16.59	2.54	13.63
	exub	5.38	4.44	3.92	1.84	23.31	2.44	11.83
negative tail	expl	4.50	4.21	1.72	2.28	7.67	2.45	7.40
	rev	5.48	4.80	3.22	1.98	14.19	2.19	11.85

of underlying stochastic processes which have finite variance.

6.2 Monte Carlo simulations

This subsection features Monte Carlo simulation results regarding the empirical power of popular tests. Based on our main findings, we construct data generating processes (DGPs) which include the stylized facts of explosive and reverse periods. In contrast to the well-known data generating process of e.g. Evans (1991), our processes are able to model a long-lasting reverse rather than a one period reverse which in most cases cannot be empirically confirmed. Moreover, they do not rely on the assumption of a full collapse, i.e. a sudden or disturbing collapse. In particular, we are able to model a smooth collapse.

First, we start with a 'plain vanilla DGP' which is later increased in its complexity. The DGP is split into one part which models the explosive behaviour of the exuberance and one part which captures the reverse period. p_0, our initial real log-price is set based on empirical observations. All following prices p_t with $t \in \{1, 2, \ldots, T_e\}$ - where T_e is the duration of the explosive period - are calculated based on an AR(1) process. The same is true for the reverse period. So, both parts of the DGP are defined as:

$$p_t = \rho_e p_{t-1} + \sigma_e u_t, \qquad t = 1, ..., T_e \tag{13}$$

$$p_t = \rho_r p_{t-1} + \sigma_r u_t, \qquad t = T_e + 1, ..., T. \tag{14}$$

We have $\rho_e > 1$ and $\rho_r < 1$. The innovations u_t are drawn from a standardized t-distributed with ν degrees of freedom. A variance shift is captured by the structural break in the scaling parameters σ_e and σ_r. We start (and end) with a random walk regime of 50 observations which has the same innovation variance as the proceeding (preceeding) explosive (mean-reverting) regime. The respective end (starting) value is matched to with the simulated trajectory in order to exclude artificial jumps in the simulated data.

We run the SADF test of Phillips, Wu, and Yu (2011) with zero lag augmentation. Hence, we provide the test with the information about the first-order lag structure and the maximum of one single explosive phase. To do so, we implement 729 different value combinations (3^6) of the parameters stated in Table 8. However, we do not impose further simplifications. In further considerations, currently under investigation, we remove these information.

Table 11: Empirical power - ceteris paribus analysis for group 1

p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
4.51	81	28	5	0.00100805	0.00127795	1.00016253	0.999804	0.051
6.76	81	28	5	0.00100805	0.00127795	1.00016253	0.999804	0.065
4.51	226	99	5	0.00100805	0.00127795	1.00016253	0.999804	0.044
4.51	81	28	5	0.00015589	0.00032906	1.00016253	0.999804	0.072
4.51	81	28	5	0.00100805	0.00127795	1.00035159	0.999804	0.058
4.51	81	28	5	0.00100805	0.00127795	1.00016253	0.999480	0.056
8.94	81	28	5	0.00100805	0.00127795	1.00016253	0.999804	0.068
4.51	815	350	5	0.00100805	0.00127795	1.00016253	0.999804	0.084
4.51	81	28	5	0.00005670	0.00009570	1.00016253	0.999804	0.080
4.51	81	28	5	0.00100805	0.00127795	1.00202689	0.999804	0.206
4.51	81	28	5	0.00100805	0.00127795	1.00016253	0.998638	0.060

Some results are already available: Empirical power for the average case (group 2 for all relevant parameters) leads to an empirical power of 52.9%. A larger starting value leads to higher power. Clearly, a longer duration of the explosive regime increases power. Also, the empirical power rises (as expected) with an increasing explosive autoregressive coefficient. Finally, an increased innovation variance reduces power as the signal-to-noise ratio worsens. The degrees of freedom ν controlling

Table 12: Empirical power - ceteris paribus analysis for group 2 (baseline)

p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
6.76	226	99	5	0.00015589	0.00032906	1.00035159	0.999480	0.529
4.51	226	99	5	0.00015589	0.00032906	1.00035159	0.999480	0.263
8.94	226	99	5	0.00015589	0.00032906	1.00035159	0.999480	0.771
6.76	81	28	5	0.00015589	0.00032906	1.00035159	0.999480	0.105
6.76	815	350	5	0.00015589	0.00032906	1.00035159	0.999480	0.910
6.76	226	99	5	0.00005670	0.00009570	1.00035159	0.999480	0.901
6.76	226	99	5	0.00100805	0.00127795	1.00035159	0.999480	0.099
6.76	226	99	5	0.00015589	0.00032906	1.00016253	0.999408	0.141
6.76	226	99	5	0.00015589	0.00032906	1.00202689	0.999480	1.000
6.76	226	99	5	0.00015589	0.00032906	1.00035159	0.998638	0.533
6.76	226	99	5	0.00015589	0.00032906	1.00035159	0.999804	0.517

Table 13: Empirical power - ceteris paribus analysis for group 3

p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
8.94	815	350	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
6.76	815	350	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	226	99	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	815	350	5	0.00015589	0.00032906	1.00202689	0.998638	1.000
8.94	815	350	5	0.00005670	0.00009570	1.00035159	0.998638	1.000
8.94	815	350	5	0.00005670	0.00009570	1.00202689	0.999480	1.000
4.51	815	350	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	81	28	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	815	350	5	0.00100805	0.00127795	1.00202689	0.998638	1.000
8.94	815	350	5	0.00005670	0.00009570	1.00016253	0.998638	0.859
8.94	815	350	5	0.00005670	0.00009570	1.00202689	0.999804	1.000

index/asset	p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
Gold (GFC)	5.65	376	127	5	0.000102	0.000324	1.000233	0.999604	0.323
NASDAQ (dot-com)	6.18	1368	386	5	0.000181	0.000914	1.000196	0.999625	0.533
Housing (GFC)	4.95	127	75	5	0.000060	0.000154	1.000396	0.999565	0.667
Oil	3.94	45	21	5	0.000413	0.000539	1.001300	0.998218	0.209

the excess kurtosis does not impact power in any noticeable way, therefore, no results are reported here but they are available on request. Results are reported in Table 12.

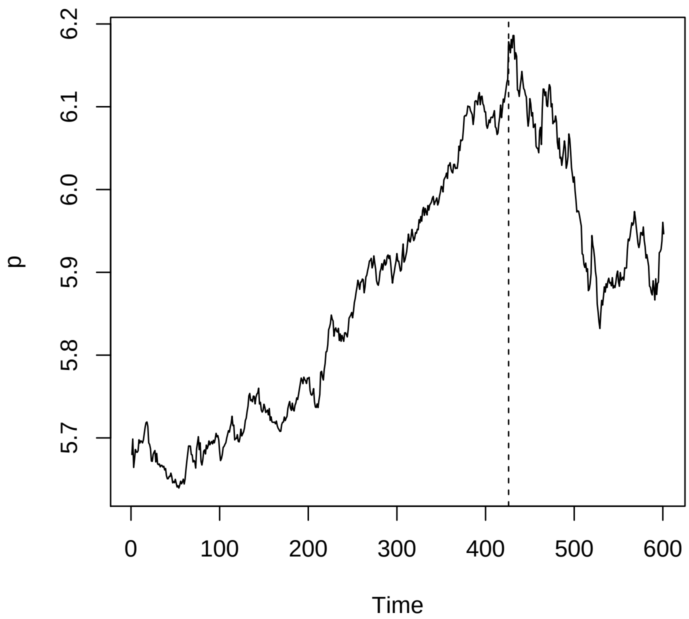

Figure 3: Typical trajectory of explosive Gold prices during the GFC.

Now, we also provide two examples which will serve throughout the analysis as 'running examples':

The first one is the Gold price during the Great Financial Crisis (GFC). The second one is the prominent Nasdaq 100 index during the famous dot-com bubble. Typical trajectories of the simulated prices are given in the two figures below. The simulated power shows that the Gold bubble was more difficult to detect in relative terms when compared to the Nasdaq, given the differences in empirical power. Overall, the empirical power is not extensively high for these two given empirical cases. As the simulations already contain a number of simplifications, the simulated power can be seen as some kind of an upper bound as the power will be reduced by search for the optimal lag length via the BIC and especially when considering a supremum statistic with an unknown timing of the bubble. Furthermore, the possibility of having more than one single explosive phase in the

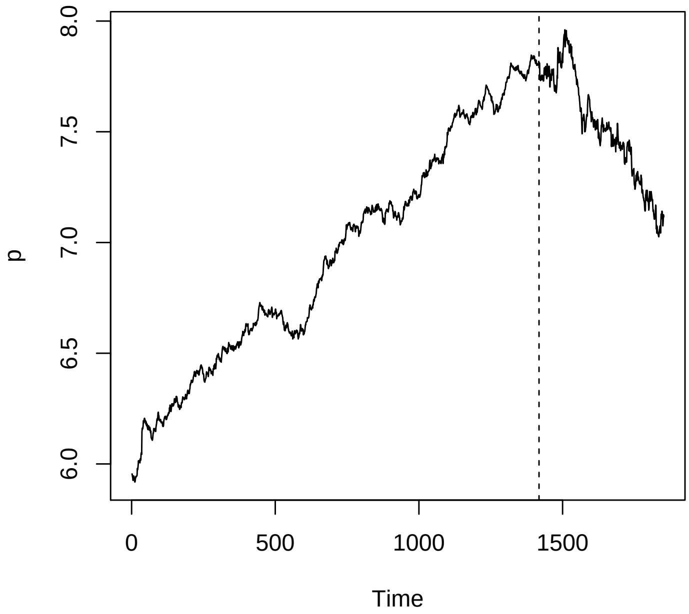

Figure 4: Typical trajectory of explosive Nasdaq prices during the dot-com bubble.

sample further reduces power. These issues are currently under consideration by the authors as well.

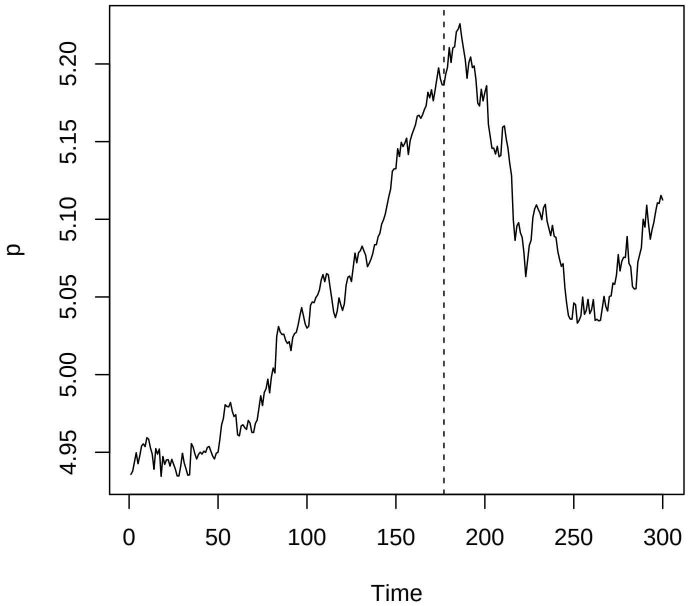

Figure 5: Typical trajectory of explosive housing prices during the GFC.

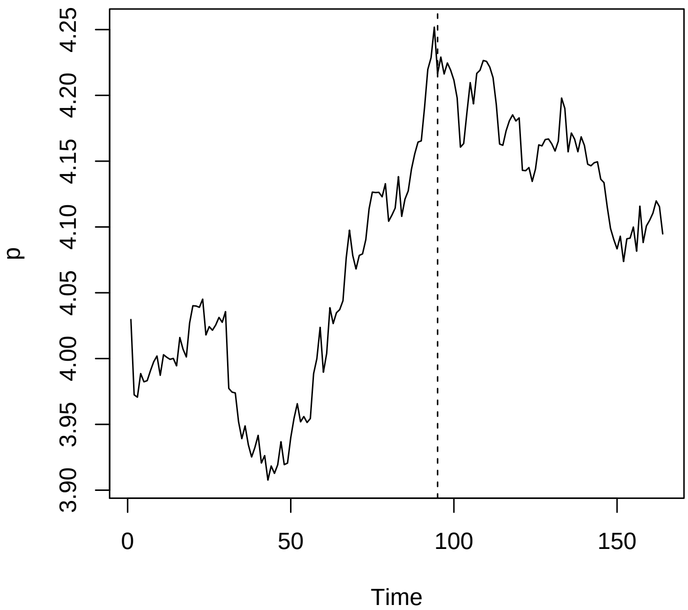

Figure 6: Typical trajectory of explosive oil prices during the GFC.

7 Conclusions

We provide new and comprehensive evidence on explosive phases in market prices. We establish a couple of new stylized facts which are useful for the understanding of these outstanding periods, often interpreted as bubbles. They are also useful for the specification of empirically relevant parameter settings in simulations for the performance of popular tests and monitoring procedures. In fact, our results on thirty different markets over a period of fifty years yields 143 explosive phases. Their characteristics deviate significantly from commonly entertained data generating processes in the following ways: (i) explosiveness is typically remarkably mild, (ii) collapse phases are smooth and (iii) market prices do not fully drop back to the initial pre-explosive value, but stay way above. We also find quite heterogeneous durations of explosive and mean-reverting phases. In addition, we confirm the existence of volatility shifts in the innovations, with an increased variance in the collapse period. Moreover, innovation distributions are fat-tailed and almost symmetric. However, the idealistic view that market prices are strongly exploding and fully collapsing in very short time is definitely not supported by our results. These stylized facts have important implications for the empirical power of tests and the performance of monitoring procedures typically conducted in practice. In particular, we find that most explosive phases of financial exuberance are much harder to detect than expected, at least given what the literature has suggested so far.

A natural extension of our research is the investigation of the multivariate perspective because during major financial exuberances, many different indices are influenced. This can e.g., be seen for the dot-com-bubble or the global financial crisis 2007/09. In the context of regulation and the subsequent avoidance of spillover and contagion effects, a knowledge of the behaviour of multiple time series (portfolio context) is essential. On top of this, the financial exuberance monitoring technique of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) could likely be enhanced by including other macroeconomic variables or by applying more advanced techniques like deep neural networks.

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Appendix

Table 14: Positive financial exuberance characteristics based on log real prices - part I

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
AEX	1985-11-19	5.25	1986-01-08	5.4	1986-02-19	5.32	67	37	30	1.23	0.15	-0.08
AEX	1993-10-05	5.52	1994-01-31	5.72	1994-03-31	5.63	128	85	43	1.98	0.2	-0.09
AEX	1995-12-27	5.78	1996-06-06	5.93	1996-07-23	5.85	150	117	33	3.55	0.15	-0.09
AEX	1996-07-25	5.84	2000-08-24	6.83	2001-03-26	6.56	1218	1066	152	7.01	0.99	-0.27
AEX	2005-12-01	6.22	2006-04-21	6.32	2006-05-11	6.3	116	102	14	7.29	0.1	-0.02
BITCOIN	2012-07-13	1.2	2012-08-16	1.76	2012-10-24	1.59	74	25	49	0.51	0.56	-0.17
BITCOIN	2013-01-08	1.77	2013-12-04	6.19	2014-09-17	5.25	442	237	205	1.16	4.42	-0.94
BITCOIN	2017-03-27	6.06	2017-12-18	8.95	2018-11-28	7.42	438	191	247	0.77	2.89	-1.52
BITCOIN	2019-05-02	7.65	2019-06-26	8.53	2019-11-21	7.99	146	40	106	0.38	0.88	-0.53
Brent Oil	2008-05-02	3.94	2008-07-03	4.18	2008-08-01	4.04	66	45	21	2.14	0.24	-0.14
CAC 40	1998-01-28	8.29	1998-07-17	8.63	1998-09-16	8.47	166	123	43	2.86	0.35	-0.16
CAC 40	1998-12-21	8.49	2000-09-04	9.06	2001-03-09	8.8	580	446	134	3.33	0.57	-0.26
CAC 40	2006-01-26	8.62	2006-05-09	8.69	2006-05-12	8.66	77	74	3	24.67	0.07	-0.03
DAX 30	1983-03-10	7.01	1983-07-07	7.18	1983-08-29	7.1	123	86	37	2.32	0.17	-0.08
DAX 30	1983-09-19	7.11	1984-02-03	7.24	1984-02-21	7.19	112	100	12	8.33	0.12	-0.05
DAX 30	1985-05-06	7.29	1986-04-17	7.87	1987-10-23	7.67	645	249	396	0.63	0.59	-0.2
DAX 30	1989-07-14	7.81	1989-09-08	7.88	1989-10-13	7.83	66	41	25	1.64	0.07	-0.05
DAX 30	1989-12-04	7.86	1990-07-18	8.04	1990-08-03	7.96	175	163	12	13.58	0.18	-0.08
DAX 30	1997-01-08	8.23	2000-03-07	9.22	2001-10-11	8.66	1242	825	417	1.98	0.99	-0.56
DAX 30	2001-11-05	8.67	2002-01-04	8.77	2002-02-05	8.7	67	45	22	2.05	0.1	-0.08
DAX 30	2006-01-25	8.74	2006-05-09	8.86	2006-05-16	8.81	80	75	5	15	0.12	-0.05
DAX 30	2007-03-29	8.96	2007-06-20	9.11	2008-01-17	9.01	211	60	151	0.4	0.15	-0.1
DJ US RE	1993-01-13	4.36	1993-03-31	4.46	1993-06-03	4.41	102	56	46	1.22	0.09	-0.05
DJ US RE	1993-06-30	4.41	1993-10-07	4.52	1993-11-03	4.47	91	72	19	3.79	0.11	-0.05
DJ US RE	1996-08-15	4.51	1997-10-07	4.81	1998-06-04	4.69	471	299	172	1.74	0.3	-0.12
DJ US RE	2003-11-24	4.62	2004-04-01	4.76	2004-04-05	4.7	96	94	2	47	0.14	-0.06
DJ US RE	2006-08-15	4.95	2007-02-07	5.2	2007-05-23	5.03	202	127	75	1.69	0.25	-0.17
FTSE SA	2005-05-26	10.2	2007-10-11	10.89	2008-01-15	10.73	689	621	68	9.13	0.69	-0.16
FTSE 100	1997-04-29	8.76	1997-10-03	8.93	1997-10-27	8.83	130	114	16	7.12	0.17	-0.1
FTSE 100	1997-12-01	8.85	1998-07-20	9.07	1998-08-27	8.93	194	166	28	5.93	0.22	-0.14

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 15: Positive financial exuberance characteristics based on log real prices - part II

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
FTSE 100	1998-12-16	8.97	1999-07-06	9.13	1999-08-09	9.05	169	145	24	6.04	0.16	-0.08
Gold	2005-12-22	5.54	2006-05-11	5.87	2006-10-03	5.67	204	101	103	0.98	0.33	-0.21
Gold	2006-10-09	5.65	2008-03-17	6.16	2008-09-10	5.86	503	376	127	2.96	0.51	-0.3
Gold	2008-12-10	5.95	2011-09-05	6.73	2013-10-10	6.32	1262	714	548	1.3	0.78	-0.41
Gold	2020-04-03	6.45	2020-08-06	6.67	2020-11-23	6.56	167	90	77	1.17	0.22	-0.11
Hang Seng	1986-10-07	8.76	1987-09-30	9.3	1987-10-23	9.13	274	257	17	15.12	0.55	-0.17
Hang Seng	1992-04-22	9.13	1992-07-16	9.29	1992-08-18	9.21	85	62	23	2.7	0.16	-0.08
Hang Seng	1993-10-06	9.45	1994-01-04	9.86	1994-03-17	9.59	117	65	52	1.25	0.41	-0.26
Hang Seng	2003-10-02	9.68	2004-02-18	9.87	2004-03-19	9.78	122	100	22	4.55	0.19	-0.09
Hang Seng	2006-10-05	10.08	2007-02-22	10.24	2007-03-02	10.15	107	101	6	16.83	0.16	-0.09
Hang Seng	2007-03-19	10.14	2007-10-30	10.62	2008-01-21	10.33	221	162	59	2.75	0.47	-0.29
HRMS	1996-09-13	6.35	1997-02-24	7.04	1998-04-21	6.67	418	117	301	0.39	0.69	-0.37
HRMS	2003-04-29	5.98	2008-01-11	8.22	2009-07-17	7.42	1624	1229	395	3.11	2.24	-0.8
HRMS	2009-07-21	7.45	2009-10-26	7.58	2009-11-04	7.48	77	70	7	10	0.13	-0.1
HRMS	2016-11-10	7.2	2017-01-17	7.36	2017-02-22	7.29	75	49	26	1.88	0.16	-0.08
HRMS	2018-01-08	7.41	2018-04-26	7.57	2018-05-31	7.48	104	79	25	3.16	0.16	-0.09
IDX	2003-11-21	7.31	2004-04-27	7.57	2004-05-07	7.46	121	113	8	14.12	0.26	-0.1
IDX	2004-10-22	7.58	2005-07-28	7.85	2005-08-22	7.74	217	200	17	11.76	0.26	-0.1
IDX	2006-09-14	7.91	2007-12-11	8.49	2008-07-02	8.24	470	324	146	2.22	0.58	-0.25
IDX	2010-09-15	8.48	2010-11-10	8.59	2011-01-07	8.54	83	41	42	0.98	0.11	-0.05
IDX	2011-03-23	8.52	2011-07-27	8.67	2011-08-08	8.58	99	91	8	11.38	0.15	-0.09
IDX	2013-02-14	8.69	2013-05-20	8.81	2013-06-07	8.73	82	68	14	4.86	0.12	-0.08
ITA 125	1992-06-19	5.82	1993-02-08	6.23	1993-07-22	6.03	285	167	118	1.42	0.41	-0.2
ITA 125	1993-07-26	6.03	1994-01-14	6.31	1994-02-18	6.17	150	125	25	5	0.27	-0.14
ITA 125	1999-11-12	6.36	2000-03-03	6.67	2000-04-04	6.5	103	81	22	3.68	0.31	-0.17
ITA 125	2003-10-15	6.34	2004-07-05	6.61	2004-08-03	6.54	210	189	21	9	0.28	-0.07
ITA 125	2004-11-11	6.57	2006-05-09	7	2006-07-12	6.85	435	389	46	8.46	0.43	-0.15
ITA 125	2006-07-14	6.83	2007-10-31	7.25	2008-03-06	7.06	430	339	91	3.73	0.41	-0.19
ITA 125	2015-03-16	7.24	2015-04-13	7.31	2015-06-25	7.28	74	21	53	0.4	0.07	-0.03
KOSPI	1986-01-31	6.23	1989-03-31	7.91	1991-04-11	7.29	1355	826	529	1.56	1.69	-0.63

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 16: Positive financial exuberance characteristics based on log real prices - part III

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
KOSPI	1999-05-31	7.03	1999-07-09	7.37	1999-09-28	7.23	87	30	57	0.53	0.34	-0.14
KOSPI	2005-11-02	7.34	2006-01-16	7.49	2006-02-14	7.42	75	54	21	2.57	0.15	-0.07
KOSPI	2007-08-20	7.65	2007-10-31	7.82	2007-11-20	7.72	67	53	14	3.79	0.17	-0.1
MASI	2016-10-28	9.26	2017-01-10	9.47	2017-03-31	9.35	111	53	58	0.91	0.21	-0.12
MASI	2017-06-16	9.38	2017-09-13	9.45	2017-12-04	9.42	122	64	58	1.1	0.08	-0.03
MASI	2018-01-12	9.43	2018-02-02	9.49	2018-05-09	9.45	84	16	68	0.24	0.05	-0.04
Mexico IPC	1989-05-18	8.34	1990-07-25	8.97	1990-08-22	8.78	330	310	20	15.5	0.63	-0.19
Mexico IPC	1991-03-08	8.86	1992-03-26	9.68	1992-08-20	9.36	380	275	105	2.62	0.82	-0.32
Mexico IPC	1993-11-16	9.66	1994-02-08	9.93	1994-03-11	9.79	84	61	23	2.65	0.27	-0.14
Mexico IPC	2003-08-18	9.53	2007-07-06	10.84	2008-12-05	10.27	1385	1015	370	2.74	1.31	-0.57
Mexico IPC	2009-11-04	10.63	2010-04-15	10.75	2010-05-17	10.68	139	117	22	5.32	0.12	-0.07
Mexico IPC	2010-09-09	10.7	2010-12-31	10.85	2011-05-05	10.76	171	82	89	0.92	0.15	-0.09
Mexico IPC	2012-12-03	10.87	2013-01-28	10.95	2013-03-14	10.88	74	41	33	1.24	0.08	-0.07
MOEX	2005-11-02	6.06	2006-05-08	6.59	2006-06-12	6.3	159	134	25	5.36	0.53	-0.29
MOEX	2006-06-19	6.31	2006-08-16	6.51	2006-09-22	6.38	70	43	27	1.59	0.21	-0.14
MOEX	2006-09-26	6.38	2007-12-12	6.66	2008-01-17	6.55	343	317	26	12.19	0.28	-0.11
NASDAQ	1978-07-10	5.2	1978-09-13	5.34	1978-10-18	5.26	73	48	25	1.92	0.14	-0.09
NASDAQ	1982-12-17	5.46	1983-06-24	5.8	1983-10-21	5.62	221	136	85	1.6	0.34	-0.18
NASDAQ	1985-11-21	5.65	1986-07-03	5.93	1986-09-15	5.75	213	161	52	3.1	0.28	-0.18
NASDAQ	1986-09-17	5.75	1986-12-04	5.8	1986-12-22	5.76	69	57	12	4.75	0.05	-0.04
NASDAQ	1987-01-05	5.78	1987-08-26	5.99	1987-10-16	5.86	205	168	37	4.54	0.2	-0.12
NASDAQ	1991-12-27	6.02	1992-02-12	6.14	1992-04-01	6.07	69	34	35	0.97	0.13	-0.08
NASDAQ	1992-11-10	6.09	1993-01-26	6.21	1993-04-23	6.13	119	56	63	0.89	0.11	-0.08
NASDAQ	1993-04-27	6.12	1994-01-31	6.31	1994-06-22	6.18	302	200	102	1.96	0.19	-0.13
NASDAQ	1994-07-07	6.17	1994-09-15	6.26	1994-12-07	6.2	110	51	59	0.86	0.09	-0.06
NASDAQ	1994-12-14	6.18	2000-03-10	7.99	2001-09-03	6.92	1754	1368	386	3.54	1.81	-1.07
NASDAQ	2017-02-10	7.76	2018-08-29	8.08	2018-12-19	7.88	484	404	80	5.05	0.31	-0.2
NASDAQ	2019-01-04	7.89	2020-02-19	8.24	2020-03-11	8.03	309	294	15	19.6	0.35	-0.21
NIFTY 500	2014-05-12	8.46	2014-09-08	8.61	2014-10-15	8.57	113	86	27	3.19	0.15	-0.04
NIFTY 500	2014-10-20	8.57	2015-03-03	8.72	2015-04-23	8.65	134	97	37	2.62	0.15	-0.06

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 17: Positive financial exuberance characteristics based on log real prices - part IV

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
NIKKEI 225	1972-03-16	9.07	1973-01-24	9.6	1973-04-24	9.36	289	225	64	3.52	0.53	-0.24
NIKKEI 225	1983-06-22	9.29	1983-10-12	9.35	1983-10-21	9.33	88	81	7	11.57	0.07	-0.02
NIKKEI 225	1983-12-01	9.34	1984-05-04	9.5	1984-05-17	9.42	121	112	9	12.44	0.16	-0.08
NIKKEI 225	1984-10-04	9.44	1989-12-29	10.68	1991-07-05	10.1	1762	1367	395	3.46	1.24	-0.58
NIKKEI 225	2005-10-31	9.55	2006-04-07	9.8	2006-05-22	9.7	146	115	31	3.71	0.26	-0.11
NIKKEI 225	2013-01-30	9.36	2013-05-22	9.69	2013-06-12	9.53	96	81	15	5.4	0.34	-0.16
NIKKEI 225	2015-02-20	9.82	2015-06-24	9.94	2015-08-20	9.9	130	89	41	2.17	0.12	-0.04
OMXH	1993-03-25	7.25	1994-02-04	7.92	1995-03-01	7.78	505	227	278	0.82	0.68	-0.14
OMXH	1995-04-25	7.78	1995-09-14	8.07	1995-10-20	7.9	129	103	26	3.96	0.29	-0.17
OMXH	1996-09-12	7.97	2000-03-06	10.06	2001-07-10	9.12	1259	908	351	2.59	2.1	-0.94
OMXH	2001-11-02	9.15	2002-01-04	9.32	2002-02-19	9.19	78	46	32	1.44	0.17	-0.12
OMXH	2005-12-02	9.15	2006-04-21	9.36	2006-05-17	9.25	119	101	18	5.61	0.21	-0.11
OMXH	2007-04-03	9.39	2007-07-13	9.51	2007-08-09	9.46	93	74	19	3.89	0.13	-0.06
Silver	2008-01-08	2	2008-03-05	2.27	2008-04-28	2.07	80	42	38	1.11	0.27	-0.2
Silver	2010-11-17	2.47	2011-04-29	3.07	2011-09-23	2.66	223	118	105	1.12	0.6	-0.41
SMI	1993-10-05	7.96	1994-01-31	8.19	1994-03-01	8.08	106	85	21	4.05	0.23	-0.11
SMI	1996-02-28	8.22	1996-04-26	8.31	1996-05-31	8.28	68	43	25	1.72	0.09	-0.03
SMI	1996-11-01	8.32	1998-07-21	9.13	1998-10-01	8.7	500	448	52	8.62	0.81	-0.43
SMI	1998-10-09	8.69	1999-01-06	9.04	2000-03-10	8.91	371	64	307	0.21	0.35	-0.12
SMI	2000-03-16	8.95	2000-08-23	9.1	2001-02-15	9.04	241	115	126	0.91	0.16	-0.07
SMI	2005-10-31	8.88	2006-03-22	9.02	2006-05-16	8.98	142	103	39	2.64	0.14	-0.03
S&P TSX	1978-12-19	8.12	1979-10-05	8.39	1979-10-19	8.27	219	209	10	20.9	0.27	-0.12
S&P TSX	1979-11-09	8.25	1980-02-29	8.55	1980-03-26	8.35	99	81	18	4.5	0.3	-0.21
S&P TSX	1980-05-14	8.36	1980-11-28	8.56	1981-01-30	8.47	188	143	45	3.18	0.2	-0.1
S&P TSX	1983-04-20	8.31	1983-09-26	8.4	1983-10-21	8.32	133	114	19	6	0.09	-0.08
S&P TSX	1987-01-27	8.51	1987-08-13	8.69	1987-10-14	8.59	187	143	44	3.25	0.18	-0.1
S&P TSX	1993-05-18	8.41	1994-03-23	8.59	1994-04-15	8.51	239	222	17	13.06	0.19	-0.08
S&P TSX	1996-03-11	8.63	1996-05-31	8.68	1996-07-12	8.64	90	60	30	2	0.06	-0.04
S&P TSX	1996-08-08	8.64	1998-04-22	9.06	1998-08-25	8.83	534	445	89	5	0.42	-0.23
S&P TSX	1999-10-28	8.93	2000-09-01	9.38	2001-02-15	9.13	341	222	119	1.87	0.45	-0.25

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 18: Positive financial exuberance characteristics based on log real prices - part V

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
S&P TSX	2005-07-05	9.15	2005-10-03	9.24	2005-10-17	9.19	75	65	10	6.5	0.09	-0.05
S&P TSX	2005-11-02	9.19	2006-04-19	9.34	2006-06-09	9.25	158	121	37	3.27	0.15	-0.09
S&P TSX	2006-10-12	9.28	2007-10-31	9.48	2008-01-18	9.34	332	275	57	4.82	0.2	-0.14
S&P TSX	2008-03-24	9.36	2008-05-20	9.48	2008-07-14	9.38	81	42	39	1.08	0.13	-0.1
S&P 500	1986-02-03	5.28	1986-08-27	5.44	1986-09-10	5.41	158	148	10	14.8	0.17	-0.03
S&P 500	1987-01-05	5.42	1987-08-25	5.68	1987-10-15	5.55	204	167	37	4.51	0.26	-0.13
S&P 500	1995-06-13	5.86	1996-05-24	6.07	1996-07-12	6.02	284	249	35	7.11	0.21	-0.05
S&P 500	1996-07-31	6.01	2000-03-24	6.79	2001-09-27	6.35	1347	953	394	2.42	0.78	-0.45
S&P 500	2001-10-02	6.38	2001-12-05	6.5	2002-04-24	6.41	147	47	100	0.47	0.11	-0.09
SSA	2006-09-08	7.45	2007-10-16	8.7	2008-03-26	8.16	404	288	116	2.48	1.25	-0.54
SSA	2014-11-03	7.83	2015-06-12	8.58	2015-07-07	8.25	177	160	17	9.41	0.75	-0.33
Straits	2006-10-19	8.12	2007-10-11	8.46	2007-11-21	8.3	285	256	29	8.83	0.35	-0.16
TOPIX	1972-02-07	6.43	1973-01-24	7.06	1973-09-04	6.8	412	253	159	1.59	0.63	-0.26
TOPIX	1983-06-16	6.67	1983-09-28	6.74	1983-10-21	6.71	92	75	17	4.41	0.07	-0.02
TOPIX	1983-10-27	6.71	1989-12-18	8.08	1991-11-29	7.5	2112	1603	509	3.15	1.37	-0.58
TOPIX	2005-09-15	7.22	2006-04-07	7.52	2006-06-05	7.4	188	147	41	3.59	0.3	-0.11
TOPIX	2013-01-30	6.88	2013-05-22	7.19	2013-06-06	7.01	92	81	11	7.36	0.31	-0.18
TOPIX	2015-02-18	7.31	2015-08-10	7.43	2015-08-20	7.39	132	124	8	15.5	0.12	-0.04
TUNINDEX	1999-08-23	7.61	2000-07-24	7.85	2000-11-17	7.79	325	241	84	2.87	0.24	-0.06
TUNINDEX	2005-04-05	7.7	2010-09-30	8.88	2013-05-27	8.52	2125	1433	692	2.07	1.18	-0.35
TUNINDEX	2015-04-07	8.6	2015-06-24	8.66	2015-08-13	8.63	93	57	36	1.58	0.07	-0.04
TUNINDEX	2017-06-15	8.6	2017-08-30	8.67	2017-09-26	8.63	74	55	19	2.89	0.06	-0.03
TUNINDEX	2018-02-12	8.66	2018-07-31	8.88	2018-10-10	8.75	173	122	51	2.39	0.22	-0.12

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 19: Negative financial exuberance characteristics based on log real prices - part I

	start date	start value	peak date	peak value	burst date	burst value	duration	duration shock	duration recov.	duration ratio	decrease shock	increase recov.
AEX	2003-01-15	6.01	2003-03-12	5.58	2003-06-05	5.86	102	41	61	0.67	-0.43	0.29
AEX	2008-09-23	6.01	2009-03-09	5.39	2009-05-07	5.64	163	120	43	2.79	-0.62	0.25
Brent Oil	2008-10-15	3.43	2008-12-26	2.78	2009-03-18	3.06	111	53	58	0.91	-0.65	0.28
Brent Oil	2014-09-30	3.68	2016-01-20	2.4	2016-05-13	2.97	424	342	82	4.17	-1.29	0.58
DAX 30	1973-06-28	7.2	1973-10-08	7.08	1973-10-18	7.12	81	73	8	9.12	-0.12	0.04
DAX 30	2002-12-05	8.26	2003-03-12	7.88	2003-04-16	8.13	95	70	25	2.8	-0.39	0.25
DJ US RE	1998-07-24	4.64	1998-10-08	4.35	1999-04-15	4.42	190	55	135	0.41	-0.29	0.07
DJ US RE	1999-07-26	4.42	1999-12-14	4.23	2000-04-12	4.34	188	102	86	1.19	-0.19	0.11
Gold	1989-03-28	5.76	1989-06-13	5.67	1989-07-04	5.71	71	56	15	3.73	-0.09	0.04
Gold	1989-07-13	5.71	1989-09-15	5.65	1989-10-26	5.68	76	47	29	1.62	-0.06	0.03
Gold	1997-03-04	5.42	1999-09-01	5.02	1999-09-27	5.12	670	652	18	36.22	-0.4	0.1
Gold	2000-07-27	5.08	2001-04-02	4.97	2001-05-18	5.04	212	178	34	5.24	-0.11	0.06
Hang Seng	1982-09-28	8.22	1982-12-02	7.81	1983-01-11	8.05	76	48	28	1.71	-0.4	0.23
HRMS	1995-11-30	6.33	1996-07-09	6.12	1996-07-18	6.17	166	159	7	22.71	-0.21	0.05
HRMS	1998-05-01	6.63	1998-12-10	6.35	1999-01-21	6.48	190	160	30	5.33	-0.29	0.13
HRMS	2000-07-19	6.29	2000-10-12	5.91	2000-11-10	6.2	83	62	21	2.95	-0.38	0.28
HRMS	2001-01-30	6.14	2002-01-28	5.64	2003-02-03	5.94	525	260	265	0.98	-0.5	0.3
HRMS	2011-08-05	7.12	2011-12-28	6.83	2012-02-09	7.05	135	104	31	3.35	-0.28	0.22
HRMS	2015-08-12	7.16	2016-01-21	6.79	2016-03-14	6.97	154	117	37	3.16	-0.37	0.17
HRMS	2018-09-14	7.26	2018-10-25	7.1	2019-01-29	7.22	98	30	68	0.44	-0.16	0.12
HRMS	2020-02-24	7.05	2020-03-18	6.63	2020-08-20	6.92	129	18	111	0.16	-0.42	0.29
IDX	1997-10-14	8.12	1998-01-09	7.59	1998-01-19	7.84	70	64	6	10.67	-0.53	0.25
IDX	1998-04-23	7.74	1998-09-21	6.81	1999-01-06	7.28	185	108	77	1.4	-0.93	0.47
IDX	2008-09-09	8.03	2008-10-28	7.46	2009-03-20	7.65	139	36	103	0.35	-0.57	0.2
ITA 125	2008-09-11	6.87	2008-11-21	6.37	2009-03-24	6.57	139	52	87	0.6	-0.5	0.2
KOSPI	1979-02-01	6.68	1979-07-13	6.4	1979-08-28	6.53	149	117	32	3.66	-0.28	0.14
KOSPI	1979-12-18	6.48	1981-01-07	5.92	1981-06-08	6.17	385	277	108	2.56	-0.56	0.25
KOSPI	1997-09-23	6.98	1997-12-12	6.33	1998-01-29	6.7	93	59	34	1.74	-0.65	0.37
KOSPI	1998-03-05	6.72	1998-06-16	6.08	1998-11-18	6.48	185	74	111	0.67	-0.64	0.41
MASI	2008-11-07	9.5	2009-01-08	9.25	2009-02-20	9.38	76	45	31	1.45	-0.25	0.13
MASI	2012-12-12	9.23	2013-08-29	9.07	2013-10-23	9.15	226	187	39	4.79	-0.16	0.07
MASI	2018-06-25	9.38	2018-11-02	9.27	2018-12-03	9.31	116	95	21	4.52	-0.11	0.04

This table provides a detailed overview of the identified negative financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the shock part, the recovery period and the complete financial exuberance period are given. The stated duration ratio is the duration of the shock part divided by the recovery. Last, the decrease and increase during both periods are given.

Table 20: Negative financial exuberance characteristics based on log real prices - part II

	start date	start value	peak date	peak value	burst date	burst value	duration	duration shock	duration recov.	duration ratio	decrease shock	increase recov.
Mexico IPC	1995-02-01	9.43	1995-03-08	9.09	1995-05-19	9.26	78	26	52	0.5	-0.34	0.18
MOEX	2008-08-19	6.2	2008-11-21	5.21	2009-03-20	5.58	154	69	85	0.81	-0.99	0.37
NASDAQ	1973-04-16	5.57	1973-07-05	5.41	1973-09-26	5.5	118	59	59	1	-0.16	0.09
NASDAQ	1973-10-30	5.49	1974-10-03	4.68	1975-06-26	5.08	433	243	190	1.28	-0.82	0.41
NASDAQ	1975-07-22	5.06	1975-12-09	4.9	1976-01-09	5	124	101	23	4.39	-0.16	0.1
NASDAQ	1990-08-16	5.72	1990-10-16	5.5	1990-12-19	5.63	90	44	46	0.96	-0.23	0.13
NIKKEI 225	1974-08-02	9.1	1974-10-09	8.79	1975-02-12	8.97	139	49	90	0.54	-0.31	0.18
NIKKEI 225	1992-03-26	9.95	1992-08-18	9.61	1992-08-27	9.82	111	104	7	14.86	-0.33	0.2
NIKKEI 225	2008-09-30	9.33	2008-10-27	8.88	2009-01-02	9.11	69	20	49	0.41	-0.45	0.23
OMXH	1989-09-21	7.95	1992-09-07	6.65	1993-03-01	7.2	898	773	125	6.18	-1.3	0.55
OMXH	2008-09-04	9.08	2009-03-06	8.42	2009-05-01	8.73	172	132	40	3.3	-0.66	0.31
S&P TSX	1974-06-11	8.36	1974-12-09	8.03	1975-02-13	8.22	178	130	48	2.71	-0.33	0.19
S&P TSX	1975-09-03	8.17	1975-11-03	8.06	1976-01-19	8.14	99	44	55	0.8	-0.12	0.08
S&P TSX	1976-10-04	8.09	1976-11-30	7.97	1977-07-13	8.04	203	42	161	0.26	-0.12	0.07
S&P TSX	1977-07-26	8.03	1977-11-01	7.93	1977-12-28	8.01	112	71	41	1.73	-0.1	0.08
S&P TSX	1978-01-02	8.01	1978-03-01	7.94	1978-04-05	8	68	43	25	1.72	-0.07	0.05
S&P TSX	1981-12-01	8.26	1982-07-08	7.8	1982-10-11	8.01	225	158	67	2.36	-0.46	0.22
S&P TSX	1990-08-21	8.39	1990-10-16	8.24	1991-02-05	8.32	121	41	80	0.51	-0.15	0.08
S&P 500	1974-03-28	5.29	1974-10-03	4.8	1975-03-14	5.08	252	136	116	1.17	-0.49	0.28
S&P 500	1978-01-06	4.99	1978-03-06	4.92	1978-04-13	4.96	70	42	28	1.5	-0.07	0.04
SSA	2008-08-08	7.87	2008-10-29	7.46	2008-12-04	7.64	85	59	26	2.27	-0.41	0.18
Straits	2008-09-23	7.97	2009-03-09	7.45	2009-04-02	7.67	138	120	18	6.67	-0.52	0.22
TOPIX	1973-12-03	6.64	1973-12-18	6.5	1974-05-28	6.57	127	12	115	0.1	-0.14	0.06
TOPIX	1974-06-21	6.56	1974-10-09	6.2	1975-03-27	6.39	200	79	121	0.65	-0.36	0.19
TOPIX	1992-02-13	7.42	1992-08-18	7.05	1993-03-10	7.24	280	134	146	0.92	-0.37	0.19
TOPIX	2008-09-30	7	2009-03-12	6.58	2009-04-08	6.73	137	118	19	6.21	-0.42	0.15
TUNINDEX	2002-10-22	7.56	2003-03-18	7.43	2003-04-15	7.51	126	106	20	5.3	-0.13	0.08
WTI Oil	1986-01-16	3.09	1986-03-31	2.26	1986-05-15	2.67	86	53	33	1.61	-0.83	0.41
WTI Oil	2014-10-31	3.52	2015-03-17	2.91	2015-04-15	3.17	119	98	21	4.67	-0.61	0.26
WTI Oil	2015-07-06	3.09	2015-08-24	2.77	2015-10-05	2.97	66	36	30	1.2	-0.32	0.2
WTI Oil	2015-11-05	2.95	2016-02-11	2.4	2016-03-08	2.73	89	71	18	3.94	-0.54	0.33

This table provides a detailed overview of the identified negative financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the shock part, the recovery period and the complete financial exuberance period are given. The stated duration ratio is the duration of the shock part divided by the recovery. Last, the decrease and increase during both periods are given.

Table 21: Basic characteristics of positive financial exuberance periods based on log real prices (duration < 1 year, duration ratio < 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	118.08	97.50	51.94	66.00	241.00	67.00	221.00
explosive duration	65.96	55.50	33.40	16.00	162.00	26.75	125.60
reverse duration	52.12	42.50	30.20	19.00	151.00	21.35	105.65
duration ratio	1.51	1.34	0.76	0.24	2.89	0.40	2.73
increase explosive	0.21	0.15	0.16	0.05	0.88	0.06	0.53
decrease reverse	-0.12	-0.08	0.10	-0.53	-0.03	-0.28	-0.03

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 22: Basic characteristics of positive financial exuberance periods based on log real prices (duration < 1 year, duration ratio ≥ 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	134.35	121.50	46.60	67.00	239.00	76.10	214.80
explosive duration	114.75	107.50	41.08	53.00	222.00	66.65	193.95
reverse duration	19.60	17.50	11.52	2.00	52.00	5.55	42.35
duration ratio	8.41	5.51	7.38	3.10	47.00	3.19	18.66
increase explosive	0.20	0.17	0.12	0.05	0.75	0.07	0.32
decrease reverse	-0.10	-0.09	0.06	-0.33	-0.02	-0.19	-0.03

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 23: Basic characteristics of positive financial exuberance periods based on log real prices (duration $\in [1, 2)$ years, duration ratio < 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	397.29	408.00	66.40	285.00	503.00	296.05	482.20
explosive duration	232.43	239.00	82.13	64.00	376.00	98.45	342.20
reverse duration	164.86	136.50	73.24	84.00	307.00	95.70	303.10
duration ratio	1.73	1.81	0.87	0.21	2.96	0.33	2.90
increase explosive	0.98	0.54	1.20	0.19	4.42	0.22	3.43
decrease reverse	-0.38	-0.26	0.40	-1.52	-0.06	-1.14	-0.10

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 24: Basic characteristics of positive financial exuberance periods based on log real prices (duration $\in [1, 2)$ years, duration ratio ≥ 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	357.92	331.00	81.94	274.00	500.00	279.50	491.20
explosive duration	313.58	302.00	69.42	225.00	448.00	238.20	423.80
reverse duration	44.33	40.50	25.08	15.00	91.00	16.10	84.95
duration ratio	9.38	8.54	5.18	3.52	19.60	3.64	17.34
increase explosive	0.42	0.38	0.18	0.20	0.81	0.21	0.71
decrease reverse	-0.19	-0.18	0.09	-0.43	-0.05	-0.33	-0.08

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 25: Basic characteristics of positive financial exuberance periods based on log real prices (duration ≥ 2 years, duration ratio < 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	1236.11	1262.00	464.20	505.00	2125.00	561.00	1829.00
explosive duration	794.44	826.00	374.56	227.00	1433.00	235.80	1265.80
reverse duration	441.67	396.00	125.85	278.00	692.00	307.20	634.40
duration ratio	1.79	1.98	0.76	0.63	2.74	0.71	2.68
increase explosive	1.12	0.99	0.51	0.59	2.10	0.63	1.94
decrease reverse	-0.47	-0.45	0.24	-0.94	-0.14	-0.82	-0.16

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 26: Basic characteristics of positive financial exuberance periods based on log real prices (duration ≥ 2 years, duration ratio ≥ 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	1284.12	1421.00	617.26	534.00	2112.00	550.10	1989.50
explosive duration	1018.12	1147.50	454.58	445.00	1603.00	445.35	1520.75
reverse duration	266.00	269.00	172.25	68.00	509.00	75.35	469.10
duration ratio	4.72	3.50	2.23	3.11	9.13	3.12	8.39
increase explosive	1.17	1.11	0.63	0.42	2.24	0.47	2.09
decrease reverse	-0.49	-0.42	0.32	-1.07	-0.16	-0.98	-0.18

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 27: Basic characteristics of positive financial exuberance periods based on log nominal prices

	mean	median	sd	min	max	5%Q	95%Q
duration	415.97	176.00	559.12	66.00	2990.00	70.00	1667.00
explosive duration	297.73	120.00	422.82	18.00	2399.00	40.00	1291.80
reverse duration	118.24	49.00	178.91	2.00	1096.00	7.60	451.40
duration ratio	5.09	2.75	6.67	0.24	54.50	0.56	15.76
increase explosive	0.48	0.26	0.60	0.05	4.43	0.07	1.58
decrease reverse	-0.20	-0.13	0.21	-1.50	-0.01	-0.57	-0.04

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 28: Positive financial exuberance characteristics based on log nominal prices - part I

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl	decrease crash
AEX	1985-11-06	9.28	1986-01-08	9.45	1986-02-21	9.35	78	46	32	1.44	0.16	-0.09
AEX	1993-09-30	9.67	1994-01-31	9.9	1994-05-31	9.81	174	88	86	1.02	0.23	-0.09
AEX	1995-07-06	9.9	1995-09-15	9.97	1995-10-20	9.93	77	52	25	2.08	0.07	-0.04
AEX	1995-11-10	9.93	2000-09-04	11.16	2001-08-30	10.87	1515	1257	258	4.87	1.22	-0.29
AEX	2005-11-18	10.64	2006-04-21	10.77	2006-05-12	10.73	126	111	15	7.4	0.14	-0.05
AEX	2007-04-04	10.85	2007-07-16	10.94	2007-07-25	10.91	81	74	7	10.57	0.08	-0.03
BITCOIN	2012-07-13	6.63	2012-08-16	7.2	2012-10-24	7.04	74	25	49	0.51	0.57	-0.16
BITCOIN	2013-01-08	7.21	2013-12-04	11.64	2014-09-17	10.72	442	237	205	1.16	4.43	-0.92
BITCOIN	2017-03-27	11.55	2017-12-18	14.45	2018-11-28	12.95	438	191	247	0.77	2.9	-1.5
BITCOIN	2019-04-30	13.17	2019-06-26	14.07	2019-11-21	13.54	148	42	106	0.4	0.9	-0.53
Brent Oil	2008-02-14	9.17	2008-07-03	9.57	2008-09-08	9.22	148	101	47	2.15	0.41	-0.35
CAC 40	1997-01-14	12.39	1997-03-10	12.51	1997-04-21	12.44	70	40	30	1.33	0.12	-0.07
CAC 40	1997-06-02	12.47	1997-10-03	12.64	1997-10-24	12.56	105	90	15	6	0.17	-0.08
CAC 40	1998-01-16	12.6	1998-07-17	12.99	1998-09-18	12.76	176	131	45	2.91	0.39	-0.24
CAC 40	1998-10-30	12.77	2000-09-04	13.45	2001-07-09	13.13	702	482	220	2.19	0.68	-0.32
CAC 40	2005-12-01	13.05	2006-05-09	13.18	2006-05-16	13.14	119	114	5	22.8	0.14	-0.04
DAX 30	2008-03-18	13.37	2008-05-19	13.49	2008-06-30	13.37	75	45	30	1.5	0.12	-0.12
DAX 30	2015-01-20	13.84	2015-04-10	14.03	2015-08-18	13.9	151	59	92	0.64	0.19	-0.13
DAX 30	2017-09-11	14.04	2018-01-23	14.12	2018-02-02	14.06	105	97	8	12.12	0.08	-0.06
DAX 30	1983-03-02	11.01	1984-02-03	11.31	1984-06-13	11.23	336	243	93	2.61	0.3	-0.09
DAX 30	1984-09-11	11.24	1986-04-17	11.97	1987-10-28	11.65	817	418	399	1.05	0.73	-0.33
DAX 30	1988-09-14	11.72	1990-03-30	12.19	1990-09-21	11.88	528	403	125	3.22	0.47	-0.31
DAX 30	1991-03-28	11.93	1991-06-11	12.05	1991-08-16	12.02	102	54	48	1.12	0.12	-0.04
DAX 30	1992-01-10	11.99	1992-05-25	12.11	1992-07-21	12.02	138	97	41	2.37	0.11	-0.09
DAX 30	1993-07-08	12.09	1994-05-16	12.33	1995-03-27	12.18	448	223	225	0.99	0.24	-0.15
DAX 30	1995-04-03	12.17	2000-03-07	13.6	2002-07-12	12.93	1900	1287	613	2.1	1.43	-0.67
DAX 30	2005-11-21	13.16	2006-05-09	13.33	2006-06-07	13.23	143	122	21	5.81	0.17	-0.1
DAX 30	2006-07-19	13.22	2007-07-16	13.61	2008-03-14	13.38	433	259	174	1.49	0.38	-0.23
DJ US RE	1993-01-11	9.3	1993-10-07	9.5	1993-11-17	9.43	223	194	29	6.69	0.2	-0.07
DJ US RE	1996-08-01	9.54	1997-10-07	9.9	1998-06-15	9.74	488	309	179	1.73	0.35	-0.16

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 29: Positive financial exuberance characteristics based on log nominal prices - part II

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl	decrease crash
DJ US RE	2003-11-03	9.83	2004-04-01	9.99	2004-04-05	9.94	111	109	2	54.5	0.16	-0.06
DJ US RE	2004-08-20	9.95	2004-12-22	10.1	2005-03-21	10.02	152	89	63	1.41	0.15	-0.08
DJ US RE	2005-04-18	10.02	2005-08-02	10.2	2005-10-11	10.07	127	77	50	1.54	0.18	-0.13
DJ US RE	2005-10-21	10.08	2007-02-07	10.52	2007-07-26	10.25	460	339	121	2.8	0.44	-0.27
FTSE SA	2005-05-20	14.11	2008-05-22	15.02	2008-09-15	14.76	867	785	82	9.57	0.91	-0.26
FTSE 100	1997-04-15	12.97	1998-07-20	13.33	1998-08-31	13.17	360	330	30	11	0.37	-0.16
FTSE 100	1998-10-29	13.19	1999-07-06	13.4	1999-09-15	13.32	230	179	51	3.51	0.21	-0.09
FTSE 100	2006-01-25	13.25	2006-04-21	13.33	2006-05-11	13.31	77	63	14	4.5	0.07	-0.01
Gold	2003-11-04	10.54	2004-01-12	10.66	2004-03-02	10.58	86	50	36	1.39	0.11	-0.08
Gold	2005-09-07	10.7	2011-09-05	12.15	2015-11-17	11.59	2660	1564	1096	1.43	1.45	-0.57
Gold	2016-01-06	11.6	2016-07-06	11.83	2016-12-14	11.66	246	131	115	1.14	0.23	-0.16
Gold	2016-12-21	11.64	2018-01-25	11.82	2018-08-10	11.71	428	287	141	2.04	0.18	-0.11
Hang Seng	1986-09-23	12.2	1987-10-01	12.89	1987-10-23	12.73	284	268	16	16.75	0.68	-0.16
Hang Seng	1992-01-17	13.01	1992-11-12	13.38	1992-12-02	13.2	229	215	14	15.36	0.37	-0.18
Hang Seng	1992-12-04	13.17	1994-01-04	14.01	1995-01-20	13.5	556	283	273	1.04	0.84	-0.52
Hang Seng	1995-01-25	13.49	1997-08-07	14.33	1997-10-29	13.89	721	662	59	11.22	0.83	-0.44
Hang Seng	1999-11-15	14.19	2000-03-28	14.42	2000-04-14	14.29	110	97	13	7.46	0.23	-0.13
Hang Seng	2000-06-05	14.28	2000-07-21	14.4	2000-09-15	14.3	75	35	40	0.88	0.12	-0.1
Hang Seng	2003-10-02	13.96	2004-02-18	14.15	2004-03-19	14.06	122	100	22	4.55	0.19	-0.09
Hang Seng	2006-07-27	14.34	2007-10-30	14.97	2008-01-21	14.68	388	329	59	5.58	0.63	-0.28
HRMS	1996-08-28	8.96	1997-02-24	9.71	1999-01-12	9.19	620	129	491	0.26	0.75	-0.52
HRMS	2003-04-08	8.82	2008-05-02	11.55	2012-06-18	10.69	2400	1324	1076	1.23	2.72	-0.85
HRMS	2012-06-25	10.74	2012-09-26	11	2013-06-11	10.81	252	68	184	0.37	0.26	-0.2
HRMS	2013-07-01	10.79	2014-09-05	11.46	2015-12-29	11.03	652	310	342	0.91	0.66	-0.43
HRMS	2016-07-08	11.14	2018-04-26	12.07	2020-06-23	11.52	1033	470	563	0.83	0.93	-0.54
IDX	2003-09-02	10.89	2004-04-27	11.31	2004-05-14	11.19	184	171	13	13.15	0.42	-0.12
IDX	2004-08-27	11.22	2008-01-09	12.55	2009-03-19	11.81	1190	879	311	2.83	1.33	-0.75
IDX	2009-05-19	12.15	2015-04-07	13.22	2015-10-05	12.98	1665	1536	129	11.91	1.07	-0.24
IDX	2016-06-29	13.12	2016-10-04	13.21	2016-11-14	13.15	99	70	29	2.41	0.09	-0.07
IDX	2016-12-28	13.16	2018-02-19	13.41	2018-04-25	13.32	346	299	47	6.36	0.25	-0.1

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 30: Positive financial exuberance characteristics based on log nominal prices - part III

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl	decrease crash
ITA 125	2015-03-18	11.84	2015-04-13	11.91	2015-06-25	11.88	72	19	53	0.36	0.07	-0.03
ITA 125	1989-04-06	8.19	1989-06-23	8.37	1989-07-07	8.31	67	57	10	5.7	0.19	-0.06
ITA 125	1989-10-19	8.4	1990-01-11	8.61	1990-01-23	8.51	69	61	8	7.62	0.21	-0.1
ITA 125	1990-04-06	8.57	1990-07-23	8.88	1990-08-08	8.69	89	77	12	6.42	0.31	-0.19
ITA 125	1991-01-22	8.84	1994-01-14	10.16	1994-06-01	9.81	877	779	98	7.95	1.32	-0.35
ITA 125	1997-05-13	10.18	1997-08-06	10.35	1997-08-28	10.27	78	62	16	3.88	0.17	-0.08
ITA 125	1999-11-01	10.64	2000-03-03	10.98	2000-10-11	10.83	248	90	158	0.57	0.34	-0.14
ITA 125	2003-10-28	10.76	2004-07-05	11.01	2004-08-03	10.93	201	180	21	8.57	0.25	-0.07
ITA 125	2004-11-11	10.96	2007-10-31	11.69	2008-03-13	11.45	871	775	96	8.07	0.72	-0.23
ITA 125	2008-04-01	11.46	2008-06-17	11.55	2008-07-02	11.48	67	56	11	5.09	0.1	-0.08
KOSPI	1977-09-02	9.36	1978-08-10	9.65	1978-09-07	9.58	265	245	20	12.25	0.28	-0.07
KOSPI	1986-01-27	9.66	1989-04-03	11.52	1992-06-19	10.95	1670	831	839	0.99	1.86	-0.57
KOSPI	1992-10-29	11.01	1993-01-11	11.17	1993-03-04	11.04	91	53	38	1.39	0.16	-0.13
KOSPI	1993-03-12	11.05	1994-11-08	11.64	1995-12-15	11.41	721	433	288	1.5	0.59	-0.24
KOSPI	1999-05-27	11.19	1999-07-09	11.54	1999-09-29	11.37	90	32	58	0.55	0.35	-0.17
KOSPI	2005-11-01	11.69	2006-05-11	11.89	2006-05-22	11.8	145	138	7	19.71	0.21	-0.09
KOSPI	2007-04-09	11.92	2007-10-31	12.24	2008-01-15	12.07	202	148	54	2.74	0.32	-0.17
MASI	2016-10-27	13.85	2017-01-10	14.07	2017-03-29	13.97	110	54	56	0.96	0.22	-0.1
MASI	2017-06-16	13.98	2017-09-13	14.06	2017-12-20	14.02	134	64	70	0.91	0.08	-0.04
MASI	2017-12-25	14.02	2018-03-09	14.1	2018-05-18	14.05	105	55	50	1.1	0.08	-0.05
Mexico IPC	1989-04-21	10.13	1992-06-01	12.16	1992-08-21	11.87	871	812	59	13.76	2.03	-0.29
Mexico IPC	1992-10-20	11.95	1993-01-06	12.11	1993-01-27	12.07	72	57	15	3.8	0.16	-0.05
Mexico IPC	1993-11-12	12.26	1994-02-08	12.57	1994-04-01	12.39	101	63	38	1.66	0.31	-0.18
Mexico IPC	1997-06-03	12.91	1997-10-21	13.19	1997-10-24	13.11	104	101	3	33.67	0.28	-0.09
Mexico IPC	2003-08-13	13.51	2007-10-18	15	2008-11-03	14.54	1364	1092	272	4.01	1.49	-0.47
Mexico IPC	2009-07-15	14.75	2011-01-05	15.17	2011-08-03	15.05	536	386	150	2.57	0.42	-0.12
Mexico IPC	2011-12-20	15.1	2012-05-07	15.2	2012-05-15	15.15	106	100	6	16.67	0.1	-0.05
Mexico IPC	2012-06-19	15.17	2013-01-28	15.34	2013-03-18	15.26	195	160	35	4.57	0.17	-0.07
MOEX	2002-03-13	10.23	2002-05-20	10.48	2002-06-18	10.37	70	49	21	2.33	0.24	-0.1
MOEX	2003-07-22	10.64	2003-10-17	10.97	2003-10-29	10.82	72	64	8	8	0.33	-0.16

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 31: Positive financial exuberance characteristics based on log nominal prices - part IV

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl	decrease crash
MOEX	2003-11-24	10.76	2004-04-12	11.14	2004-05-10	10.93	121	101	20	5.05	0.38	-0.21
MOEX	2005-07-18	11.12	2007-12-12	12.19	2008-07-10	12.02	779	628	151	4.16	1.07	-0.17
NASDAQ	1978-04-07	9.29	1978-09-13	9.54	1978-10-25	9.4	144	114	30	3.8	0.25	-0.14
NASDAQ	1979-03-01	9.42	1980-02-08	9.71	1980-03-21	9.54	277	247	30	8.23	0.29	-0.17
NASDAQ	1980-05-05	9.55	1981-05-29	10.01	1982-03-04	9.78	479	280	199	1.41	0.46	-0.24
NASDAQ	1982-09-03	9.81	1987-08-26	10.73	1987-12-02	10.33	1369	1299	70	18.56	0.92	-0.4
NASDAQ	1987-12-14	10.34	1989-10-09	10.79	1990-09-03	10.55	711	476	235	2.03	0.45	-0.24
NASDAQ	1991-01-28	10.59	2000-03-10	13.13	2001-09-05	12.08	2768	2380	388	6.13	2.54	-1.05
NASDAQ	2001-11-05	12.1	2002-01-04	12.24	2002-02-05	12.12	67	45	22	2.05	0.14	-0.11
NASDAQ	2018-07-05	13.54	2018-08-29	13.61	2018-10-09	13.56	69	40	29	1.38	0.07	-0.05
NIFTY 500	2014-05-09	13.19	2015-03-03	13.51	2015-09-01	13.39	343	213	130	1.64	0.31	-0.12
NIFTY 500	2016-07-01	13.46	2016-09-08	13.55	2016-11-10	13.51	95	50	45	1.11	0.08	-0.04
NIFTY 500	2017-01-25	13.52	2018-08-31	13.81	2019-02-15	13.69	538	418	120	3.48	0.3	-0.12
NIFTY 500	2019-02-20	13.69	2019-06-03	13.81	2019-07-26	13.73	113	74	39	1.9	0.11	-0.08
NIFTY 500	2019-10-17	13.75	2020-01-17	13.83	2020-02-25	13.79	94	67	27	2.48	0.07	-0.04
NIKKEI 225	1972-01-28	12.56	1973-01-24	13.19	1973-11-12	13.02	467	259	208	1.25	0.63	-0.17
NIKKEI 225	1980-08-08	13.43	1981-02-02	13.5	1981-03-11	13.47	154	127	27	4.7	0.08	-0.04
NIKKEI 225	1981-03-16	13.48	1981-08-17	13.59	1981-09-25	13.51	140	111	29	3.83	0.12	-0.09
NIKKEI 225	1981-10-29	13.52	1982-01-27	13.58	1982-02-26	13.53	87	65	22	2.95	0.06	-0.05
NIKKEI 225	1983-03-02	13.59	1989-12-29	15.17	1991-08-16	14.64	2208	1783	425	4.2	1.58	-0.53
NIKKEI 225	2005-10-31	14.12	2006-04-07	14.38	2006-05-22	14.28	146	115	31	3.71	0.26	-0.1
NIKKEI 225	2013-01-30	13.92	2013-05-22	14.26	2013-06-12	14.1	96	81	15	5.4	0.34	-0.16
NIKKEI 225	2015-02-12	14.4	2015-06-24	14.55	2015-08-21	14.48	137	95	42	2.26	0.15	-0.07
OMXH	1993-03-18	11.53	1994-02-04	12.19	1995-03-07	12.02	514	232	282	0.82	0.67	-0.17
OMXH	1995-04-24	12.05	1995-09-14	12.36	1995-11-15	12.14	148	104	44	2.36	0.31	-0.22
OMXH	1996-08-07	12.21	2000-05-02	14.42	2001-08-28	13.4	1320	975	345	2.83	2.21	-1.02
OMXH	2001-10-11	13.46	2002-01-04	13.71	2002-04-18	13.52	136	62	74	0.84	0.26	-0.19
OMXH	2005-12-01	13.57	2006-04-21	13.8	2006-05-19	13.67	122	102	20	5.1	0.23	-0.13
OMXH	2007-03-15	13.81	2007-07-13	13.98	2007-08-15	13.91	110	87	23	3.78	0.17	-0.07
OMXH	2007-08-17	13.89	2007-11-07	14.05	2007-12-14	13.97	86	59	27	2.19	0.16	-0.09

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 32: Positive financial exuberance characteristics based on log nominal prices - part V

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl	decrease crash
Silver	2006-02-16	6.84	2006-05-11	7.31	2006-06-12	7.02	83	61	22	2.77	0.47	-0.29
Silver	2006-10-09	7.03	2007-02-23	7.29	2007-06-25	7.17	186	100	86	1.16	0.26	-0.12
Silver	2007-09-19	7.17	2008-03-05	7.64	2008-08-08	7.34	233	121	112	1.08	0.47	-0.29
Silver	2010-09-23	7.66	2011-04-29	8.49	2012-06-27	7.9	460	157	303	0.52	0.83	-0.59
Silver	2012-06-29	7.92	2012-10-04	8.16	2013-03-29	7.95	196	70	126	0.56	0.24	-0.21
SMI	1993-10-01	12.42	1994-01-31	12.67	1994-04-20	12.55	144	87	57	1.53	0.25	-0.12
SMI	1996-02-28	12.73	1996-07-11	12.85	1996-07-19	12.81	103	97	6	16.17	0.13	-0.04
SMI	1996-09-24	12.81	1998-07-21	13.64	1998-09-30	13.26	527	476	51	9.33	0.83	-0.38
SMI	1998-10-12	13.26	1999-01-06	13.55	1999-10-15	13.43	265	63	202	0.31	0.29	-0.12
SMI	1999-10-19	13.44	1999-12-30	13.54	2000-01-24	13.48	70	53	17	3.12	0.1	-0.06
SMI	2000-03-21	13.5	2000-08-23	13.64	2000-09-28	13.58	138	112	26	4.31	0.14	-0.05
SMI	2005-09-07	13.41	2006-05-09	13.61	2006-05-16	13.58	180	175	5	35	0.19	-0.03
S&P TSX	1972-11-22	11.7	1973-01-12	11.77	1973-03-21	11.74	86	38	48	0.79	0.07	-0.02
S&P TSX	1978-05-12	11.61	1980-11-28	12.39	1982-01-08	12.15	956	666	290	2.3	0.78	-0.24
S&P TSX	1983-01-27	12.21	1983-09-26	12.47	1984-05-21	12.32	343	173	170	1.02	0.25	-0.15
S&P TSX	1984-08-03	12.35	1987-08-13	12.93	1987-10-23	12.64	841	790	51	15.49	0.58	-0.29
S&P TSX	1988-02-12	12.63	1988-07-05	12.76	1988-11-10	12.69	195	103	92	1.12	0.12	-0.06
S&P TSX	1988-11-21	12.7	1989-10-06	12.91	1990-04-10	12.79	362	230	132	1.74	0.21	-0.12
S&P TSX	1993-04-29	12.84	1994-03-23	13.04	1994-04-15	12.96	252	235	17	13.82	0.21	-0.08
S&P TSX	1995-12-19	13.05	1998-04-22	13.57	1998-08-26	13.33	702	612	90	6.8	0.52	-0.24
S&P TSX	1998-10-30	13.34	1999-01-08	13.44	1999-03-01	13.35	87	51	36	1.42	0.1	-0.09
S&P TSX	1999-03-05	13.37	2000-09-01	13.95	2001-03-20	13.56	533	391	142	2.75	0.57	-0.38
S&P TSX	2003-12-16	13.59	2004-03-01	13.7	2004-03-23	13.65	71	55	16	3.44	0.11	-0.05
S&P TSX	2005-05-27	13.78	2008-06-18	14.23	2008-09-05	14.06	856	799	57	14.02	0.45	-0.16
S&P TSX	2014-06-04	14.21	2014-09-03	14.26	2014-09-19	14.24	78	66	12	5.5	0.06	-0.03
S&P 500	1983-04-11	9.65	1983-06-22	9.75	1983-07-28	9.71	79	53	26	2.04	0.1	-0.04
S&P 500	1985-11-05	9.86	1987-08-25	10.42	1987-10-16	10.25	509	471	38	12.39	0.56	-0.18
S&P 500	1988-02-09	10.13	1988-04-13	10.21	1988-05-10	10.16	66	47	19	2.47	0.08	-0.05
S&P 500	1988-11-18	10.19	1990-07-16	10.52	1990-09-21	10.35	481	432	49	8.82	0.33	-0.17
S&P 500	1991-01-15	10.35	2000-03-24	11.94	2002-07-01	11.48	2990	2399	591	4.06	1.58	-0.46

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 33: Positive financial exuberance characteristics based on log nominal prices - part VI

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl	decrease crash
S&P 500	2003-12-18	11.6	2004-02-11	11.66	2004-03-19	11.62	67	40	27	1.48	0.06	-0.04
S&P 500	2006-10-04	11.81	2007-02-20	11.89	2007-03-02	11.84	108	100	8	12.5	0.08	-0.05
S&P 500	2007-03-14	11.84	2007-07-19	11.95	2007-08-13	11.89	109	92	17	5.41	0.11	-0.07
SSA	2006-09-11	12.08	2007-10-16	13.37	2008-03-26	12.84	403	287	116	2.47	1.29	-0.52
SSA	2014-11-20	12.46	2015-06-12	13.2	2015-07-07	12.88	164	147	17	8.65	0.75	-0.33
Straits	2005-06-22	12.28	2005-08-02	12.35	2005-10-14	12.31	83	30	53	0.57	0.08	-0.04
Straits	2005-12-27	12.33	2006-05-08	12.47	2006-05-24	12.38	107	95	12	7.92	0.14	-0.09
Straits	2006-08-30	12.4	2007-10-11	12.86	2008-01-18	12.65	363	292	71	4.11	0.45	-0.21
TOPIX	1972-01-07	9.92	1973-01-24	10.65	1973-11-15	10.41	485	274	211	1.3	0.73	-0.25
TOPIX	1978-08-17	10.64	1979-01-31	10.74	1979-04-09	10.69	168	120	48	2.5	0.1	-0.05
TOPIX	1980-08-12	10.76	1981-08-17	11.01	1982-07-05	10.88	495	265	230	1.15	0.25	-0.13
TOPIX	1982-10-08	10.91	1989-12-18	12.57	1992-01-10	12.02	2416	1877	539	3.48	1.66	-0.55
TOPIX	2005-09-15	11.8	2006-04-07	12.09	2006-06-06	11.96	189	147	42	3.5	0.3	-0.13
TOPIX	2013-01-30	11.45	2013-05-22	11.76	2013-06-12	11.61	96	81	15	5.4	0.31	-0.15
TOPIX	2013-10-09	11.67	2014-01-08	11.78	2014-01-29	11.74	81	66	15	4.4	0.11	-0.04
TOPIX	2015-02-12	11.88	2015-08-10	12.04	2015-08-21	11.97	137	128	9	14.22	0.15	-0.07
TUNINDEX	1999-02-03	11.54	1999-02-26	11.66	1999-06-10	11.58	92	18	74	0.24	0.12	-0.07
TUNINDEX	1999-08-04	11.6	2000-09-13	11.89	2001-01-15	11.84	379	291	88	3.31	0.29	-0.05
TUNINDEX	2004-03-19	11.76	2004-06-07	11.81	2004-08-24	11.79	113	57	56	1.02	0.05	-0.01
TUNINDEX	2005-04-01	11.84	2010-09-30	13.25	2013-12-17	12.99	2273	1435	838	1.71	1.41	-0.26
TUNINDEX	2014-10-28	13.07	2015-06-24	13.27	2015-10-19	13.16	255	172	83	2.07	0.19	-0.11
TUNINDEX	2017-06-06	13.27	2018-08-28	13.64	2019-11-20	13.45	642	321	321	1	0.38	-0.19
WTI Oil	2008-04-09	9.31	2008-07-11	9.59	2008-07-22	9.46	75	68	7	9.71	0.27	-0.13

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 34: Analysed subsamples based on log nominal prices

	duration ratio < 3	duration ratio ≥ 3	$\sum$
duration < 1 year	53	49	102
duration $\in [1, 2)$ years	20	10	30
duration > 2 years	20	21	41

Table 35: Basic characteristics of positive financial exuberance periods based on log nominal prices (duration < 1 year, duration ratio < 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	119.42	101.00	49.13	66.00	248.00	68.20	214.40
explosive duration	66.34	59.00	30.13	18.00	148.00	28.00	125.00
reverse duration	53.08	47.00	29.84	19.00	158.00	22.00	113.20
duration ratio	1.49	1.39	0.74	0.24	2.95	0.47	2.75
increase explosive	0.20	0.14	0.16	0.05	0.90	0.07	0.47
decrease reverse	-0.12	-0.09	0.09	-0.53	-0.01	-0.29	-0.04

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 36: Basic characteristics of positive financial exuberance periods based on log nominal prices (duration < 1 year, duration ratio ≥ 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	121.82	110.00	45.14	67.00	230.00	69.40	214.20
explosive duration	105.49	100.00	40.20	53.00	215.00	56.40	179.60
reverse duration	16.33	15.00	10.11	2.00	51.00	5.00	33.40
duration ratio	9.65	5.81	9.59	3.12	54.50	3.50	29.32
increase explosive	0.20	0.17	0.12	0.06	0.75	0.08	0.38
decrease reverse	-0.09	-0.08	0.06	-0.33	-0.01	-0.19	-0.03

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 37: Basic characteristics of positive financial exuberance periods based on log nominal prices (duration $\in [1, 2)$ years, duration ratio < 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	404.10	435.50	80.19	252.00	495.00	254.85	488.35
explosive duration	226.45	240.00	72.50	63.00	339.00	67.75	310.50
reverse duration	177.65	181.50	55.38	83.00	303.00	92.50	249.80
duration ratio	1.44	1.35	0.71	0.31	2.80	0.37	2.62
increase explosive	0.75	0.33	1.06	0.18	4.43	0.19	2.98
decrease reverse	-0.31	-0.16	0.35	-1.50	-0.09	-0.95	-0.11

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 38: Basic characteristics of positive financial exuberance periods based on log nominal prices (duration $\in [1, 2)$ years, duration ratio ≥ 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	339.50	353.00	70.82	252.00	481.00	257.85	439.15
explosive duration	296.80	291.50	58.01	235.00	432.00	239.50	386.10
reverse duration	42.70	38.50	24.47	16.00	88.00	16.45	80.35
duration ratio	9.02	8.53	4.39	3.31	16.75	3.67	15.43
increase explosive	0.38	0.31	0.16	0.21	0.68	0.23	0.66
decrease reverse	-0.15	-0.16	0.07	-0.28	-0.05	-0.25	-0.06

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 39: Basic characteristics of positive financial exuberance periods based on log nominal prices (duration ≥ 1 year, duration ratio < 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	1120.30	769.00	687.91	514.00	2660.00	532.05	2413.00
explosive duration	664.60	473.00	437.20	129.00	1564.00	226.85	1441.45
reverse duration	455.70	331.50	290.92	142.00	1096.00	149.60	1077.00
duration ratio	1.62	1.46	0.78	0.26	2.83	0.79	2.83
increase explosive	1.04	0.76	0.64	0.38	2.72	0.42	2.24
decrease reverse	-0.45	-0.41	0.24	-1.02	-0.12	-0.86	-0.17

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 40: Basic characteristics of positive financial exuberance periods based on log nominal prices (duration $\geq$ 1 year, duration ratio $\geq$ 3)

	mean	median	sd	min	max	5%Q	95%Q
duration	1227.71	871.00	763.74	509.00	2990.00	527.00	2768.00
explosive duration	1049.19	790.00	614.92	403.00	2399.00	418.00	2380.00
reverse duration	178.52	98.00	168.39	38.00	591.00	51.00	539.00
duration ratio	8.41	7.95	4.59	3.22	18.56	3.48	15.49
increase explosive	1.08	0.92	0.58	0.30	2.54	0.45	2.03
decrease reverse	-0.35	-0.29	0.20	-1.05	-0.12	-0.55	-0.16

This table provides an overview of the basic characteristics of the identified positive financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

Table 41: Negative financial exuberance characteristics based on log nominal prices - part I

	start date	start value	peak date	peak value	burst date	burst value	duration	duration shock	duration recov.	duration ratio	decrease shock	increase recov.
AEX	2008-09-26	10.48	2009-03-09	9.9	2009-05-01	10.09	156	117	39	3	-0.58	0.19
Brent Oil	2008-10-15	8.81	2008-12-26	8.12	2009-03-18	8.42	111	53	58	0.91	-0.68	0.29
Brent Oil	2014-10-02	9.12	2016-01-20	7.86	2016-05-12	8.44	421	340	81	4.2	-1.26	0.58
CAC 40	2009-01-12	12.69	2009-03-09	12.44	2009-04-15	12.61	68	41	27	1.52	-0.25	0.17
DAX 30	2003-01-07	12.65	2003-03-12	12.3	2003-04-14	12.53	70	47	23	2.04	-0.35	0.23
DJ US RE	1998-07-27	9.71	1998-10-08	9.45	1999-04-15	9.53	189	54	135	0.4	-0.27	0.08
DJ US RE	1999-08-03	9.52	1999-12-14	9.36	2000-01-07	9.46	114	96	18	5.33	-0.16	0.1
Gold	1997-05-15	10.45	1997-07-07	10.37	1997-09-30	10.41	99	38	61	0.62	-0.08	0.04
Gold	1997-10-09	10.4	1998-08-28	10.22	1998-10-01	10.31	256	232	24	9.67	-0.18	0.09
Gold	1998-10-09	10.3	1999-01-27	10.25	1999-03-08	10.27	107	79	28	2.82	-0.05	0.02
Gold	1999-03-12	10.28	1999-07-20	10.14	1999-09-24	10.2	141	93	48	1.94	-0.14	0.06
Hang Seng	1982-09-30	11.44	1982-12-02	11.12	1983-01-07	11.29	72	46	26	1.77	-0.32	0.17
HRMS	1995-12-19	8.93	1996-07-09	8.78	1996-07-16	8.82	151	146	5	29.2	-0.15	0.03
HRMS	2000-07-20	9.07	2000-10-12	8.71	2000-11-10	9	82	61	21	2.9	-0.35	0.28
HRMS	2001-02-08	8.92	2002-01-28	8.47	2003-01-29	8.66	515	253	262	0.97	-0.45	0.19
ITA 125	2008-09-26	11.29	2008-11-21	10.85	2009-03-24	11.06	128	41	87	0.47	-0.44	0.2
KOSPI	1997-10-15	11.01	1997-12-12	10.47	1998-01-14	10.77	66	43	23	1.87	-0.54	0.3
KOSPI	1998-04-16	10.72	1998-06-16	10.24	1998-10-19	10.53	133	44	89	0.49	-0.48	0.29
MASI	2013-06-27	13.7	2013-08-29	13.64	2013-10-03	13.67	71	46	25	1.84	-0.06	0.03
MASI	2018-06-26	14	2018-11-02	13.9	2018-11-27	13.93	111	94	17	5.53	-0.1	0.03
MOEX	2008-09-29	11.53	2008-10-24	10.85	2009-02-06	11.09	95	20	75	0.27	-0.69	0.24
NASDAQ	1973-11-14	9.23	1973-12-24	9.09	1974-03-12	9.17	85	29	56	0.52	-0.14	0.08
NASDAQ	1974-03-18	9.16	1974-10-03	8.61	1975-03-07	8.91	255	144	111	1.3	-0.55	0.3
NIKKEI 225	2008-10-02	13.92	2008-10-27	13.48	2009-01-02	13.69	67	18	49	0.37	-0.44	0.21
OMXH	1989-10-16	12.02	1989-11-23	11.89	1990-01-19	11.97	70	29	41	0.71	-0.13	0.08
OMXH	1990-03-13	11.97	1992-09-07	10.9	1993-01-06	11.39	737	650	87	7.47	-1.07	0.49
OMXH	2008-09-22	13.53	2009-03-06	12.93	2009-04-29	13.21	158	120	38	3.16	-0.6	0.28
S&P TSX	1982-01-12	12.12	1982-07-08	11.81	1982-08-25	11.96	162	128	34	3.76	-0.31	0.15
S&P TSX	1990-08-28	12.72	1990-10-16	12.61	1990-12-11	12.69	76	36	40	0.9	-0.11	0.07
S&P 500	1974-07-26	9.02	1974-10-03	8.74	1975-01-24	8.9	131	50	81	0.62	-0.28	0.16

This table provides a detailed overview of the identified negative financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the shock part, the recovery period and the complete financial exuberance period are given. The stated duration ratio is the duration of the shock part divided by the recovery. Last, the decrease and increase during both periods are given.

Table 42: Negative financial exuberance characteristics based on log nominal prices - part II

	start date	start value	peak date	peak value	burst date	burst value	duration	duration shock	duration recov.	duration ratio	decrease shock	increase recov.
SSA	2008-08-04	12.57	2008-11-04	12.1	2008-12-05	12.26	90	67	23	2.91	-0.47	0.17
Straits	2008-09-25	12.41	2009-03-09	11.89	2009-04-03	12.11	137	118	19	6.21	-0.52	0.22
TOPIX	1992-03-05	11.93	1992-08-18	11.61	1992-08-27	11.81	126	119	7	17	-0.32	0.2
TOPIX	2008-11-06	11.42	2009-03-12	11.16	2009-04-08	11.31	110	91	19	4.79	-0.26	0.15
WTI Oil	1986-01-20	7.66	1986-03-31	6.95	1986-05-07	7.33	78	51	27	1.89	-0.71	0.38
WTI Oil	2014-10-31	8.99	2015-03-17	8.38	2015-04-14	8.58	118	98	20	4.9	-0.62	0.2
WTI Oil	2015-11-06	8.4	2016-02-11	7.87	2016-03-04	8.19	86	70	16	4.38	-0.52	0.32

This table provides a detailed overview of the identified negative financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the shock part, the recovery period and the complete financial exuberance period are given. The stated duration ratio is the duration of the shock part divided by the recovery. Last, the decrease and increase during both periods are given.