International integration, diversification and banking stability.*

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ABSTRACT:

Banks are exposed globally to risk either directly through their portfolio and funding, or indirectly through their affiliates and the structure of their banking group. Therefore, it is crucial to consider the global framework in which banks operate in order to assess banking stability. In this paper, I first develop a theoretical model to focus on the impact of international integration on the resilience of banks. Starting from an accounting identity, I derive the volatility of bank’s equity as a function of asset returns, funding costs, foreign exchange rate and leverage. In particular, I ask how the variance co-variance matrix between each component of the balance sheet impacts the volatility of equity returns. This model introduces a banking theory on international diversification where three main channels are identified: the global financial cycle channel; the within channel of assets and liabilities; and the foreign exchange rate channel. In the second part of the paper, I depict the financial evolution of international integration between the US and the Euro area from 2000 to 2015, estimating conditional correlations between asset returns, funding costs and exchange rate fluctuations. Focusing on this period, the model predicts that international diversification improves the resilience of banks even during periods of strong international integration such as 2008. Finally, this exercise brings to light some common patterns between what the theoretical model predicts and what the euro area banks actually do.

JEL classification: F31, F34, F36, F37, F44, G15, G20

Keywords: bank, financial cycle, international integration, banking stability, diversification, foreign exchange rate, financial markets, risk sharing, volatility.

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1 Introduction

Banks are exposed globally to risk either directly through their portfolio and funding, or indirectly through their affiliates and the structure of their banking group. In fact the Global Financial Crisis has drawn special attention to international activities of banks, especially for European banks. With this international diversification, banks are directly exposed to financial risk and monetary policy shocks from different geographical locations. The degree of connection between financial and monetary shocks in different geographical areas then depends on the degree of international integration. Therefore, it is crucial to consider the global framework in which banks operate in order to assess banking stability and to answer the following questions. How does the global composition of bank’s balance sheet impact the resilience of banks? Under which conditions could banks make use of international integration to improve their resilience?

Following the Portfolio Theory introduced by Markowitz [1952], Grubel [1968] Levy and Sarnat [1970] and Lessard [1973] first show that international diversification is a source of welfare gains for investors. In addition to the imperfect correlation of equity returns and a limited exposure to a common element of variance, Solnik [1974] highlights the specific role of foreign currency exposure in international diversification. Because changes in parities impact the final yield of a foreign asset, correlations between equity returns and foreign currency enter the demand for foreign assets and the benefit of international diversification. There is a risk management demand for foreign currency where foreign currency is used to manage the risk of portfolio (Campbell et al. [2010]).

Considering banks, strategies are not limited to portfolios but they also include the management of debt in different geographical locations. Using the structure of their banking group and their internal capital market (BIS [2010], Cetorelli and Goldberg 2010) 1

1 See Kroencke et al. [2014] for the speculative demand of foreign currency
multinational banks define strategies to manage their funding worldwide. Doing so, banks are exposed to global financial conditions. Combining both assets and liabilities, Conovera et al. [1999], Rey [2013], Miranda-Agrippino and Rey [2015] identify a global financial cycle where the US monetary policy has a major influence on credit conditions and funding costs worldwide. All thing considered, international diversification is not limited to assets and the literature based on portfolio theory does not capture the different components of banks’ balance sheet that are potentially correlated.

The purpose of this paper is to consider the diversification of both assets and liabilities in order to introduce a banking strategy of international diversification. As international integration affects the international market linkages (Longin and Solnik [1995]), the variance covariance matrices of equity returns, funding costs and foreign currency is changing over time.\(^2\) If the benefit of asset diversification is unstable because of the asymmetry of correlations between equity returns (Longin and Solnik [1995], Ang and Bekaert [2002] and Driessen and Laeven [2007]), one may expect that the benefit of banking diversification would also depend on international integration.

In this paper, I first develop a theoretical model to focus on the impact of international integration on bank’s resilience. Starting from a balance sheet identity, I derive the volatility of bank’s equity as a function of asset returns, funding costs, foreign exchange rate and leverage. In particular, I ask how the variance co-variance matrix between each component of the balance sheet impacts the volatility of equity returns.\(^3\) Doing so, this paper introduce a banking theory on international diversification that considers

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\(^2\) See FSB [2019] and Claessens [2019] for a complete definition of international integration. In this paper, the definition of international integration focuses on the difference in prices of economically similar prices. Therefore, it follows Jappelli and Pagano [2008], Stavarek et al. [2012], Ruscher and Vasicek [2016], Horny et al. [2018], Liu et al. [2018] where the deviation of the low of one price is seen as a symptom of market fragmentation. In other words, international integration implies high correlations between financial assets that share similar properties.

\(^3\) Following Diamond and Rajan [2000], the focus on banks’ equity is determinant to banking stability as it is a buffer against financial losses.
risks on assets, liabilities and foreign currency simultaneously. In the second part of
the paper, I use bi-variate DCC-GARCH to estimate the variance co-variance matrix of
asset returns, funding costs and foreign exchange rate from 2000 to 2015. This exercise
provides crucial details on the evolution of international integration over time. Using the
estimated characteristics of international integration, I calibrate the model to identify
the level of international diversification that provides the best resilience of banks: the
efficient international diversification. Doing so, this paper provides a first comparison
between the efficient international diversification and the observed international diversi-
fication of euro area banks from 2000 to 2015.

My results identify three main channels through which international diversification
and international integration affect bank’s resilience. The global financial cycle channel
is the first channel. It captures the risk sharing capacity between asset returns or be-
tween funding costs. The intuitions behind this channel are the same as those of the
international diversification theory. Strong correlations between asset returns (funding
costs) reduce the risk sharing capacity of asset diversification (liability diversification).
The second channel is the within balance sheet channel of assets and liabilities. It con-
siders the fact that the resilience of banks depends simultaneously on shocks on asset
returns and funding costs. Therefore, strong correlations between asset returns and
funding costs introduce a natural pass-through which increases the resilience of banks.
The last channel identified in this model is the foreign exchange rate channel. It is
the most complex channel as it captures all interactions with foreign exchange rate
in both sides of the balance sheet. Whether a floating exchange rate is stabilizing or
destabilizing depends on foreign exchange rate behavior with regard to shocks on asset
returns and funding costs. Considering all channels together, the model defines a level
of international diversification that assures the highest resilience of banks considering
characteristics of international integration. In some cases, currency mismatch may offer
new hedging strategies. In fact, the calibration of the model with data from 2000 to 2015 confirms the benefits from international diversification: introducing diversification in assets and liabilities increases banks’ resilience even during periods of strong international integration such as 2008. It also provides a detailed explanation of large and efficient currency mismatch between 2009 and 2012, reflecting the US dollar shortage and pointing out the necessity to decompose equity volatility with all components. Although magnitudes differ between efficient and observed international diversification, this exercise brings to light some common patterns between what the theoretical model predicts and what the EA banks actually do. First, both observed and predicted international liability diversification share similar trends over the period. Second, international liability diversification - either observed or predicted - shows larger fluctuations than international asset diversification. Third, the period of low currency mismatch predicted by the model is verified for EA banks. All-in-all, it suggests that EA banks follow the banking strategy of international diversification to maximize their resilience.

Because international integration implies dynamic interactions between asset returns, funding costs and exchange rate, the main challenge of this research is to deliver results that are intuitive and easily identifiable. This paper meets that challenge for three reasons. First, it provides a new framework to capture all interactions. Focusing on asset returns and funding costs, I consider correlations between asset returns from different geographic locations, correlations between funding costs from different geographic locations, but also, correlations between asset returns and funding costs in a given geographic location, and finally, correlations between asset returns and funding cost across different geographic locations. Correlations with foreign exchange rate complete this global framework. Second, the definition of the model - starting from an accounting identity - provides a simple and flexible framework that allows the identification of the different channels through which international integration affects banking stability. The channels
are then intuitive and easily identifiable. Third, by using DDC-GARCH to estimate dynamic correlations between assets returns, funding costs and foreign exchange rate, I consider the fact that international integration is a moving process where interactions change over time. Doing so, this paper significantly contributes to the literature on the international transmission of shocks through international banking activity, including Milesi-Ferretti and Tille [2011], Cetorelli and Goldberg [2011], Claessens and van Horen [2015], Bruno and Shin [2015], Cerutti [2015], Pedrono [2017], Baskaya et al. [2017], Cerutti et al. [2017]. It goes beyond the portfolio theory of international diversification (Solnik [1974], Campbell et al. [2010], Driessen and Laeven [2007]) by considering both leverage and diversification opportunities on liabilities. And last but not least, this new framework provides innovative results including conditions on foreign currency behavior relative to funding costs and a new channel through which international diversification improves banking stability: the within balance sheet channel.

The remainder of the paper is organized as follows. Section 2 develops the theoretical framework based on a simplified definition of a bank’s balance sheet. The volatility of equity returns as a function of financial integration is then introduced, allowing for the definition of an efficient diversification which improves the resilience of banks. Section 3 first provides a picture of international integration from 2000 to 2015 by estimating conditional volatilities and correlations. Using the variance co-variance matrix, efficient international diversification is defined and compared to observed international diversification of EA banks over the period. Section 4 concludes.
2 Theoretical model

2.1 Definition of equity

2.1.1 Assets

Bank’s total asset $A$ is composed of domestic asset $C$ and foreign asset converted in domestic currency $SC^*$ where $S$ is the exchange rate. The share of domestic and foreign asset are given by $(1 - \psi)$ and $\psi$ respectively.

\[ A = C + SC^* \]  
\[ \frac{C}{A} = (1 - \psi) ; \quad \frac{SC^*}{A} = \psi \]

Following Solnik [1974], the exchange rate and both asset returns follow stochastic processes with marginal variations defined as:

\[ d\tilde{C} = d\frac{C}{C} = r - \sigma_C dZ_C \]  
\[ d\tilde{C}^* = d\frac{C^*}{C} = r^* - \sigma_{C^*} dZ_{C^*} \]  
\[ d\tilde{S} = d\frac{S}{S} = \mu - \sigma_S dZ_S \]

$r, r^*$ and $\mu$ are the deterministic parts of the returns, and $\sigma_C, \sigma_{C^*}$ and $\sigma_S$ are the stochastic part. White noises are denoted $dZ$ such that $dZ_C \sim N(0; dt)$, $dZ_{C^*} \sim N(0; dt)$ and $dZ_S \sim N(0; dt)$.

Using Itô’s lemma, the return of the foreign asset converted in domestic currency is

\[ \text{For more details on Stochastic Differential Equations, see Merton [1971], Branson and Henderson [1984], Oksendal [2003].} \]
defined as:

\[ dSC^* = \frac{dSC^*}{SC^*} = \frac{dS}{S} + \frac{dC^*}{C^*} + \frac{dS}{S} \cdot \frac{dC^*}{C^*} \]

\[ = (\mu + r^* + \rho_{SC^*} \sigma_S \sigma_{C^*}) dt + \sigma_S dZ_S + \sigma_{C^*} dZ_{C^*} \]  

(5)

Where \( \rho_{SC^*} \) is the coefficient of correlation between the return of the foreign asset and the foreign currency.

2.1.2 Liabilities

Bank’s total debt \( D \) consists of domestic liability \( L \) and foreign liability converted in domestic currency \( SL^* \). Denote \((1 - \lambda)\) and \(\lambda\) the share of domestic and foreign liabilities respectively.

\[ D = L + SL^* \]  

\[ \frac{L}{D} = (1 - \lambda); \quad \frac{SL^*}{D} = \lambda \]  

(6)

Introducing stochastic processes, we get the following Stochastic Differential Equations (SDE) for each liability:

\[ d\tilde{L} = \frac{dL}{L} = i dt + \sigma_L dZ_L \]  

\[ d\tilde{L}^* = \frac{dL^*}{L^*} = i^* dt + \sigma_{L^*} dZ_{L^*} \]  

(7)  

(8)

Where \( dZ_L \) and \( dZ_{L^*} \) are white noises, and \( i \) and \( i^* \) are the constant drifts of the marginal variation of domestic liability and foreign liability respectively. \( \sigma_L \) and \( \sigma_{L^*} \) are the volatility of the marginal variation of domestic liability and the foreign liability, respectively.
The foreign funding cost converted in domestic currency is defined as:

\[ d\tilde{S}L^* = \frac{dS}{S}L^* + \frac{dL}{L}S^* = (\mu + i^* + \rho_{SL^*}\sigma_S\rho_{L^*})dt + \sigma_SdZ_S + \sigma_{L^*}dZ_{L^*} \]  \hspace{1cm} (9)

Where \( \rho_{SL^*} \) is the coefficient of correlation between the foreign funding cost and the foreign currency.

### 2.1.3 Equity

Bank’s equity is defined through \( E \) such that:

\[ E = A - D \]  \hspace{1cm} (10)

Bank’s leverage \( l \) is the ratio of total debts over equity.

\[ l = D/E \]  \hspace{1cm} (11)

Following the Basel III framework, I assume that leverage is exogenous and defined by authorities. Using definitions of \( l \) and \( E \), we obtain the bank's equity SDE:
\[
\begin{align*}
\frac{d\tilde{E}}{\tilde{E}} &= (1 + l) \frac{dA}{A} - l \cdot \frac{dD}{D} \\
&= (1 + l) \left( (1 - \psi) \frac{dC}{C} + \psi \frac{dSC^*}{SC^*} \right) + l \cdot \left( (1 - \lambda) \frac{dL}{L} + \lambda \frac{dSL^*}{SL^*} \right) \\
&= [r + \psi(r^* - r) + l(\psi r^* - \lambda i^*) + l((1 - \psi) r - (1 - \lambda)i)] \, dt \\
&\quad + [\mu(\psi + l(\psi - \lambda)) + \psi(1 + l)\sigma_{SC^*} - l\lambda \sigma_{SL^*}] \, dt \\
&\quad + (1 + l) \left( (1 - \psi) \sigma_C dZ_C + \psi(\sigma_{C^*} dZ_{C^*} + \sigma_S dZ_S) \right) \\
&\quad - l \left( (1 - \lambda) \sigma_L dZ_L + \lambda(\sigma_{L^*} dZ_{L^*} + \sigma_S dZ_S) \right) \\
&= \mu(\psi + l(\psi - \lambda)) + \psi(1 + l)\sigma_{SC^*} - l\lambda \sigma_{SL^*} \] (12)
\end{align*}
\]

If \( \psi = \lambda = 0 \),

\[
\frac{d\tilde{E}}{\tilde{E}} = (1 + l) \left[ r + l(r - i) \right] \, dt + \sigma_C dZ_C + l(\sigma_C dZ_C - \sigma_L dZ_L)
\]

In absence of international diversification (e.g. \( \psi=0 \) and \( \lambda=0 \)), the marginal variation of equity does not depend on foreign components. Interestingly, if asset return and funding cost share similar properties (i.e. \( r = i \) and \( \sigma_c = \sigma_L \)), the shareholders of the bank still receive a positive return on equity because of leverage. The return on equity is equal to the return of asset.

Considering international diversification, the expected return of equity \( \text{E}(d\tilde{E}) \) includes the excess return of foreign asset, the foreign and the domestic spreads between asset returns and funding costs, and the profitability related to the foreign currency. \( \text{E}(d\tilde{E}) \) is such that:

\[
\text{E}(d\tilde{E}) = [r + \psi(r^* - r) + l(\psi r^* - \lambda i^*) + l((1 - \psi)r - (1 - \lambda)i)] \\
+ [\mu(\psi + l(\psi - \lambda)) + \psi(1 + l)\sigma_{SC^*} - l\lambda \sigma_{SL^*}] \] (13)
Where $\sigma_{SC^*}$ and $\sigma_{SL^*}$ are the covariances between the foreign currency and the foreign asset and the covariances between the foreign currency and the foreign debt, respectively. The expected equity return highlights the role of foreign currency mismatch conditional to the behavior of foreign currency relative to asset return and funding cost. If $\sigma_{SC^*}$ is negative, gains from foreign asset returns is compensated by changes in parities. The profitability of being exposed to the foreign asset is reduced. On the contrary, a negative $\sigma_{SL^*}$ increases the profitability of being diversified internationally as changes in funding costs are erased by changes in parities. Those findings are close to Campbell et al. [2010] conclusions where foreign currency exposures impact the final yield of investors. Therefore, the behavior of foreign currency relative to asset returns and funding costs changes the investment and funding opportunity set faced by banks.

In absence of international diversification, the volatility of equity is composed of the volatility of the domestic asset and liability and of the covariance between these two components. It is defined as:

$$\text{Var}(\frac{d\tilde{E}}{dt}) = (1 + l)^2 \sigma_C^2 + l^2 \sigma_L^2 - 2l(1 + l)\sigma_{LC}$$ (14)

Where $\sigma_{LC}$ is the covariance between the domestic liability and the domestic asset (i.e. $\text{Cov}(z_L, z_C)$). A positive correlation between returns on domestic asset and funding costs on domestic liability then decreases the volatility of equity: shocks on asset returns are hedged by shocks on funding costs.

2.2 Volatility of equity and international diversification

2.2.1 Volatility of equity

Assuming that shocks are not orthogonal, the volatility of equity includes all possible correlations between the different components of the bank’s balance sheet $\{C, C^*, L, L^*, S\}$. 

10
Therefore, \( \sigma_{CC\star}, \sigma_{LL\star}, \sigma_{LC}, \sigma_{L\star C\star}, \sigma_{L\star C}, \sigma_{LC\star} \), denote the covariance between assets, the covariance between liabilities, the covariance between domestic asset and liability, the covariance between foreign asset and liability, the covariance between foreign liability and domestic asset, and the covariance between the domestic liability and the foreign asset respectively. Additionally, \( \sigma_{SC\star}, \sigma_{SC}, \sigma_{SL\star} \) and \( \sigma_{SL} \) illustrate the covariance between exchange rate and foreign asset, the covariance between exchange rate and domestic asset, the covariance between exchange rate and foreign liability and the covariance between the exchange rate and domestic liability respectively. Considering all potential covariances, the variance of equity marginal variation \( \Phi^2 \) can be decomposed as:

\[
\Phi^2 = \text{Var}(\frac{d\tilde{E}}{dt}) = \left((1 + l)(1 - \psi)\sigma^2_C + (1 + l)^2\sigma^2_C\star + (\psi + l(\psi - \lambda))^2\sigma^2_S + (l(1 - \lambda))^2\sigma^2_L + (l\lambda)^2\sigma^2_{L\star C}\right)
\]

with orthogonal shocks

\[+ \frac{2(1 + l)^2\psi(1 - \psi)\sigma_{CC\star}}{\text{global financial cycle risk, asset}} + \frac{l^2\lambda(1 - \lambda)\sigma_{LL\star}}{\text{global financial cycle risk, liability}}
\]

\[- \frac{2(1 + l)l[(1 - \psi)(1 - \lambda)\sigma_{LC} + \lambda\sigma_{L\star C\star} + \psi(\lambda\sigma_{L\star C\star} + (1 - \lambda)\sigma_{LC\star})]}{A-D hedging strategies}
\]

\[+ \frac{2(\psi + l(\psi - \lambda))(1 + l)(1 - \psi)\sigma_{SC}}{\text{FX channel, asset}} + \psi\sigma_{SC\star}}
\]

\[- \frac{2(\psi + l(\psi - \lambda))l[(1 - \lambda)\sigma_{SL} + \lambda\sigma_{SL\star}]}{\text{FX channel, liability}}
\]

The first line of \( \Phi^2 \) summarizes the volatility of equity when all shocks are orthogonal. It depends positively on risks from \( C, C\star, L, L\star \) and \( S \). When \( \psi = \lambda \neq 0 \) (no currency mismatch), the exchange rate still impacts equity volatility because of the leverage multiplier: a currency match does not remove exchange rate risk. The exchange rate risk is removed if and only if \( \frac{\psi}{\lambda} = \frac{l}{1+l} \): as long as \( l > 0 \), it implies that \( \psi \neq \lambda \) when \( \psi > 0 \).
The second line of $\Phi^2$ introduces the risk added by global financial cycle on both sides of the balance sheet. A positive correlation between $C$ and $C^*$ raises the variance of equity through $\sigma_{CC^*}$: it translates the global financial cycle risk coming from the asset side and repeats the basic conditions of international portfolio diversification from Grubel [1968], Levy and Sarnat [1970] and Lessard [1973]. Similarly, the global financial cycle channel related to the liability side is introduced with the covariance $\sigma_{LL^*}$. It introduces the link through which domestic and foreign funding conditions are correlated (Rey [2013]).

The third line of $\Phi^2$ introduces hedging strategies between assets and liabilities, exchange rate movements aside: the within balance sheet channel. A positive correlation between funding costs and asset returns makes equity more resilient to shocks. Equity volatility is thus reduced by this spontaneous mechanism. Notice that domestic asset can be used to hedge foreign liability if $\sigma_{L^*C} > 0$. Similarly, foreign asset can be used to hedge domestic liability if $\sigma_{LC^*} > 0$. Therefore, equation (12) suggests that currency mismatch can reduce the volatility of equity.

The fourth line of $\Phi^2$ introduces the foreign exchange rate channel on the asset side. This channel is removed either when $\psi = \lambda = 0$, or when $\frac{\psi}{\lambda} = \frac{1}{1 + t}$. Following previous empirical studies including Andersen et al. [2003, 2007], Faust et al. [2007], Ehrmann et al. [2011], there is a negative correlation between asset returns and exchange rate, implying that $\sigma_{SC^*} > 0$ and $\sigma_{SC} < 0$ (i.e. positive shock on the return of a financial asset goes hand in hand with an appreciation of the currency attached to the financial asset). Assuming that $\sigma_{SC} = -\sigma_{SC^*}$ with $\sigma_{SC^*} > 0$, the introduction of foreign exchange rate correlations leads to different conclusions depending on international diversification. When $\psi = 0.5$, the two covariances $\sigma_{SC}$ and $\sigma_{SC^*}$ have similar weight,
translating a neutrality in exchange rate fluctuations relative to shocks on asset returns. Thus, there is no additional impact on equity volatility when $\psi = 0.5$. When $\psi > 0.5$, the weight associated to $\sigma_{SC}^*$ increases: a simultaneous and positive shock on domestic and foreign asset returns is associated with an increase in converted asset returns due to foreign currency appreciation. The FX channel on assets then amplifies equity volatility. However, with international diversification on the liability side, the foreign currency appreciation also increases converted funding costs, compensating the increase in converted asset returns due to foreign currency appreciation. When $\psi = 0.5$, fluctuations in converted funding costs completely cancel out the fluctuations in converted returns due to FX channel. When diversification on liability is large enough (i.e. $\psi < \frac{1}{1+\lambda}$), the increase in converted funding costs also compensated for the initial positive shock on asset returns, reducing equity volatility. When $\psi < 0.5$, the weight associated to $\sigma_{SC}$ increases: a positive shock on domestic and foreign asset returns is associated with a domestic currency appreciation. The latter lowers the increase in foreign asset returns: increase in total asset returns is moderate and equity volatility is stabilized. The decrease in converted funding costs implied by a relatively low diversification of liability (i.e. $\frac{\psi}{1+\lambda} > \frac{1}{1+\lambda}$) still enables a reduction in equity volatility. However, when the diversification of liabilities is relatively high (i.e. $\frac{1+\lambda}{1+\lambda} < \lambda$), then the induced decrease in converted funding costs exceeds the decrease in converted returns from foreign asset: the gap between total returns and total funding costs widens and the variance of equity increases. Thus, bank’s resilience decreases when $\frac{\psi}{1+\lambda} < \frac{1}{1+\lambda}$ and increases when $\frac{\psi}{1+\lambda} > \frac{1}{1+\lambda}$.

The last line of $\Phi^2$ introduces the foreign exchange rate channel on the liability side. Following empirical evidences, $\sigma_{SL} < 0$ and $\sigma_{SL}^* > 0$. Assuming that $\sigma_{SL} = -\sigma_{SL}^*$, with $\sigma_{SL}^* > 0$, the introduction of correlations between foreign exchange rate and fund-

\footnote{This result is line with [Campbell et al. 2010] where a negative correlation between foreign stock markets and foreign currency reduces portfolio returns volatility. However, their conclusion does not consider the implications of an international diversification on liabilities.}
ing costs also leads to different conclusions depending on international diversification. Similarly to the FX channel on assets, there is no additional impact on bank’s resilience when \( \lambda = 0.5 \). However, when \( \lambda \neq 0.5 \), the introduction of correlations relative to foreign exchange rate and funding costs impacts the volatility of equity. When \( \lambda > 0.5 \) and \( \psi \) is relatively low (i.e. \( \psi < \frac{1}{1+\lambda} \)), the introduction of correlations relative to foreign exchange rate and funding costs increases the volatility of equity. When \( \psi \) becomes relatively large (i.e. \( \psi > \frac{1}{1+\lambda} \)), the introduction of correlations relative to foreign exchange rate and funding costs decreases the volatility of equity. Conversely, when \( \lambda < 0.5 \), a relatively low \( \psi \) (i.e. \( \psi < \frac{1}{1+\lambda} \)) decreases equity volatility while a relatively large \( \psi \) (i.e. \( \psi > \frac{1}{1+\lambda} \)) increases it. Combining with the FX channel on the asset side, the leverage effect extends the impact of correlations linked to assets relative to those linked to liabilities.

### 2.2.2 Efficient International Diversification

I derive from the variance of equity marginal variation an "efficient" share of asset denominated in foreign currency \( \hat{\psi} \), where \( \hat{\psi} \) is defined so as to minimize the volatility of equity. Considering all potential correlations between each component \{\( C, C^*, L, L^*, S \}\}, the "efficient" level of asset diversification \( \hat{\psi} \) is defined such as:

\[
\text{Denote } \frac{\partial^2 \Phi^2}{\partial \psi^2} \text{ the second derivative of } \Phi^2 \text{ with respect to } \psi. \text{ Because of the non negative property of the variance } \text{Var} \left( \frac{dC^*}{dt} - \frac{dC}{dt} + \frac{dS}{dt} \right), \frac{\partial^2 \Phi^2}{\partial \psi^2} \geq 0. \text{ Therefore, } \Phi^2 \text{ is convex and } \hat{\psi} \text{ is a minimum.}
\]
\[ \frac{\partial \Phi^2}{\partial \psi} = 0 \mid \lambda \text{ constant} \]

\[ \hat{\psi} = \frac{\sigma_C^2 - \sigma_{CC^*} - \sigma_{SC}}{\sigma_C^2 + \sigma_{CC^*} + \sigma_S^2 - 2(\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ \text{share of } C \text{ in asset-side risk} \]

\[ + \lambda \left( \frac{l}{1+l} \right) \frac{\sigma_S^2 + \sigma_{SC^*} - \sigma_{SC}}{\sigma_C^2 + \sigma_{CC^*} + \sigma_S^2 - 2(\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ \text{share of } C^* \text{ to hedge FX risk due to } L^* \]

\[ + \lambda \left( \frac{l}{1+l} \right) \frac{\sigma_{SL^*} + \sigma_{L^*C^*} - \sigma_{L^*C}}{\sigma_C^2 + \sigma_{CC^*} + \sigma_S^2 - 2(\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ \text{share of } C^* \text{ risk that can be hedged with } L^* \]

\[ + (1-\lambda) \left( \frac{l}{1+l} \right) \frac{\sigma_{SL^*} + \sigma_{LC^*} - \sigma_{LC}}{\sigma_C^2 + \sigma_{CC^*} + \sigma_S^2 - 2(\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ \text{share of } C^* \text{ risk that can be hedged with } L \]

The first component of equation (16) is the share of total assets volatility driven by domestic asset volatility. The higher this share, the higher the efficient asset diversification. The second component introduces the risk reduction related to part of the liability side being also in foreign currency: it depends on \( \lambda \) and \( l \). If \( \lambda \neq 0 \) and \( \sigma_S^2 \neq 0 \), the share of assets denominated in foreign currency can be used to hedge foreign risk introduced by foreign liability. If \( \lambda=0 \) (i.e. no liability diversification), exchange rate volatility \( \sigma_S^2 \) is as important as the foreign asset volatility \( \sigma_C^2 \) in the determination of efficient asset diversification.

The last two components of equation (16) link efficient asset diversification with the international diversification of liability. First, it introduces the share of foreign asset risk that can be hedged with foreign liability. The higher the liability diversification, the larger the efficient asset diversification. Second, it introduces the share of foreign asset risk that can be hedged with domestic liability: a larger efficient asset diversification can be justified if domestic liability is a good instrument to hedge against shocks on foreign
asset (i.e. \((\sigma_{SL} + \sigma_{LC^*} - \sigma_{LC}) > 0\)). In this case, the lower the liability diversification, the larger the efficient asset diversification.

Turning on the "efficient" level of liabilities denominated in foreign currency, the complete framework with all potential correlations implies a \(\hat{\lambda}\) such that:

\[
\frac{\partial \Phi^2}{\partial \lambda} = 0 \quad | \psi \text{ constant} \\
\hat{\lambda} = \frac{\sigma_L^2 - \sigma_{LL^*} - \sigma_{SL}}{\sigma_L^2 + \sigma_{LL^*} + \sigma_{SS}^2 - 2(\sigma_{LL^*} + \sigma_{SL^*} - \sigma_{SL})}
\]

\[
\text{share of } L \text{ in liability-side risk}
\]

\[
+ \psi \left(1 + \frac{1}{l}\right) \frac{\sigma_{LL^*}^2 + \sigma_{SL^*} - \sigma_{SL}}{\sigma_L^2 + \sigma_{LL^*} + \sigma_{SS}^2 - 2(\sigma_{LL^*} + \sigma_{SL^*} - \sigma_{SL})}
\]

\[
\text{share of } L^* \text{ to hedge FX risk due to } C^*
\]

\[
+ \psi \left(1 + \frac{1}{l}\right) \frac{\sigma_{LL^*} + \sigma_{SL}^* - \sigma_{LC^*}}{\sigma_L^2 + \sigma_{LL^*} + \sigma_{SS}^2 - 2(\sigma_{LL^*} + \sigma_{SL^*} - \sigma_{SL})}
\]

\[
\text{share of } L^* \text{ risk that can be hedged with } C^*
\]

\[
+ (1 - \psi) \left(1 + \frac{1}{l}\right) \frac{\sigma_{SC} + \sigma_{LC} - \sigma_{LC^*}}{\sigma_L^2 + \sigma_{LL^*} + \sigma_{SS}^2 - 2(\sigma_{LL^*} + \sigma_{SL^*} - \sigma_{SL})}
\]

\[
\text{share of } L^* \text{ risk that can be hedged with } C
\]  

As for \(\hat{\psi}\), \(\hat{\lambda}\) decomposes each share of liability-side risk that can be hedged either with risk diversification within liabilities, or with the foreign exchange rate exposure included in asset diversification, or because foreign asset is a potential instrument to hedge against foreign funding costs, or on the contrary, because domestic asset is potentially a good instrument to hedge against risks associated to foreign funding costs.
3 International integration: an application to US and EA financial markets

3.1 Data

Defining the euro as the domestic currency and the US dollar as the foreign currency, the Eurostoxx50 and the S&P500 stock market indices are used to measure domestic and foreign assets, respectively. The euro area Shadow Short Rate (EA SSR) and the US Shadow Short Rate (US SSR) are used to identify domestic and foreign funding costs, respectively. SSR are estimations of monetary policy interest rate adjusting for unconventional monetary policy. According to Aizenman et al. [2015], they provide a good representation of liquidity availability, especially when monetary interest rates are at the zero bound. Because multinational banks have a significant share of market based operations (Geneva-Report [2019]), stock market indices and SSRs provide good measure to capture the exposures of banks to financial markets and monetary policy.

Figure 1 illustrates these four variables plus the exchange rate between US dollar and euro. Over the period 2000-2015, there is a common trend for both international stock market indexes and Shadow Short Rates, confirming the financial comovements between international stocks from Evans and McMillan [2009], Bekaert et al. [2009] and the global financial cycle from Rey [2013], Miranda-Agrippino and Rey [2015]. Concerning the foreign exchange rate, the euro appreciates against the US dollar from 2001 to 2008 while the second half of the period is characterized by more stagnation and some

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7US and euro SSR are from the Reserve Bank of New Zealand (http://www.rbnz.govt.nz/research-and-publications/research-programme/additional-research/measures-of-the-stance-of-united-states-monetary-policy/comparison-of-international-monetary-policy-measures). According to this report, the dependence on market based operations is even more important for large banks as they tend to do less traditional banking than smaller banks.

8The exchange rate is defined as the amount of euros per unit of US dollar. Daily SSRs have been extracted from annualized SSRs.
depreciation of the euro. Finally, figure 1 also suggests that international stocks have larger volatility than both the US and the EA SSR.

Table 1 reports descriptive statistics of variable first differences. The low mean of returns associated with large standard deviations and excess kurtosis translate the traditional characteristics of daily financial variables. Although figure 1 suggests some notions of correlations between variables - especially for stock market indices and SSR - specific empirical analysis is necessary to identify properly correlations and volatility. Section 4 is devoted to this exercise with estimations of bi-variate Dynamic Conditional Correlation GARCH (DCC GARCH).

3.2 Conditional variance and correlations of assets, liabilities and exchange rate

To identify variances of \{C, C^*, L, L^*, S\} and correlations between the different components, bivariate DCC GARCH(1,1) are estimated using daily data from 2000 to 2015. The log returns of the S&P500 index and the log returns of the Eurostox50 are used to proxy foreign (C^*) and domestic (C) returns respectively. Concerning funding costs, I use the US SSR changes and the EA SSR changes to proxy monetary tightening in the foreign economy (L^*) and in the domestic economy (L) respectively. Figure 1 pictures the four time series and the foreign exchange rate between the US dollar and the euro; while Table 1 displays summary statistics on each variable.

Compared to cointegration analysis, dynamic conditional variances and correlations provided by bivariate GARCH analysis capture the potential changes in financial integration as mentioned by Evans and McMillan 2009. Because international integration

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10 A definition of DCC GARCH is developed in the appendix.
11 Bloomberg is the main source for data on stock index while data on SSR are estimated by the Reserve Bank of New Zealand.
implies increasing cross-border holdings of a wide range financial assets, one may expect a convergence in related prices (Ruscher and Vasicek [2016]). Alternatively, changes in correlations between financial market indices and global funding costs reflect developments in international integration. Because risks, imperfect market conditions (transaction costs, home bias, etc.) and economic outlook are priced in asset returns, funding costs and foreign currency, this measure of international integration captures - at least partially - the different reasons of international fragmentation.\(^{12}\)

As illustrated in Figure 1, the period 2000-2015 includes different phases of financial integration starting with the introduction of the euro at the beginning of 2000’s, then followed by a systemic financial crisis in 2008-2009, to finally express an increasing divergence in international stock indexes and SSRs since 2013. In 2013, the S&P 500 exceeds its pre-crisis level and continue to increases until 2015, while the Eurostoxx50 stays below its pre-crisis level (see Figure 1a). Regarding SSR and Figure 1b, US SSR increases since 2013 while EA SSR declines.

Figure 2 illustrates conditional variances of each component while table 2 provides details on average conditional variances per year. Both figure 2 and table 2 confirm the unstable variance of financial assets as highlighted in Longin and Solnik [1995], Ang and Bekaert [2002] and Bekaert et al. [2009]. For both assets and liabilities, the period from 2004 to 2007 is characterized by low conditional variances while volatility surges in 2001-2002 and in 2008-2009, corresponding to the bursting of the dot-com bubble and the subprime crisis respectively. For each variable, US and euro area counterparts show similar volatility movements between 2004 and 2011. As reported in table 2 the euro area stock index displays larger volatility on average than the US stock index except in 2008: \(\sigma^2_C > \sigma^2_{C*}\). Regarding SSRs, the results vary more than for the stock indexes: on

\(^{12}\)See FSB [2019] for a complete definition of international fragmentation.
average, the EA SSR volatility is higher than the US SSR volatility in 2000 and 2003, during the financial crisis in 2009 and since 2011, implying that $\sigma_{L}^{2} > \sigma_{L^{*}}^{2}$. Comparing both sides of the bank’s balance sheet from 2000 to 2015, stock returns are on average more volatile than changes in SSR: $\sigma_{C}^{2} > \sigma_{L}^{2}$, $\sigma_{C}^{2} > \sigma_{L^{*}}^{2}$, $\sigma_{C}^{2} > \sigma_{L}^{2}$ and $\sigma_{C}^{2} > \sigma_{L^{*}}^{2}$.

The foreign exchange rate exhibits large increase in volatility at the beginning of the euro and during financial distresses from the end of 2008 to 2011. However, the average volatility of the exchange rate is lower than the volatility of stock returns but larger than the volatility of SSR changes over the period. The difference in volatilities between stock returns, foreign exchange rate and SSR changes translates the different degree of uncertainty for each category where stock index imply more uncertainty than monetary policy. Finally, 2015 presents several episodes of increased volatility for all component of the balance sheet except US SSR.

{ Insert Figure 2 here }

{ Insert Table 2 here }

Figure 3 displays the conditional correlations between stock returns and SSR changes. The average conditional correlations per year are reported in table 3. Over the period 2000-2015, $\rho_{LL^{*}}$ and $\rho_{CC^{*}}$ are mainly positive. However, on average $\rho_{CC^{*}}$ is higher than $\rho_{LL^{*}}$, translating a stronger financial cycle on stock returns than on SSR changes. $\rho_{LC}$ and $\rho_{L^{*}C^{*}}$ are also mainly positive over the period and they exhibit similar trends until 2010, expressing potential common behaviors among stock returns and SSR changes for a given currency area. Additionally, the similarity between shocks on stock returns and SSR changes is larger in the US and in the EA until 2013 ($\rho_{LC} \geq \rho_{L^{*}C^{*}}$). In 2014 and 2015 $\rho_{LC}$ decreases and becomes lower than $\rho_{L^{*}C^{*}}$. The last two correlations $\rho_{L^{*}C}$ and $\rho_{LC}$ suggest potential hedging strategies among assets and liabilities in different currencies. In 2003 and between 2009 and 2012, $\rho_{LC^{*}} > \rho_{L^{*}C^{*}}$, implying that domestic liability offers more hedging capability than foreign liability regarding risks on foreign
asset. Similarly, $\rho_{LC^*} > \rho_{LC}$ in 2005 and since 2014, offering some potential gains from a currency mismatch.

\[
\{ \text{Insert Figure} 3 \text{ here } \}
\]

\[
\{ \text{Insert Table} 3 \text{ here } \}
\]

Figure 4 pictures correlations associated to the exchange rate. The average conditional correlations per year are reported in table 3. One interesting fact from the estimation of conditional correlations is that the four correlations $\{\rho_{SL}, \rho_{SL^*}, \rho_{SC}, \rho_{SC^*}\}$ have similar patterns with three distinct trends. First, they are mainly increasing from 2000 to 2003. Then, they are decreasing until 2011, before increasing again after 2012. The dynamic behavior of correlations makes them switch from positive to negative value over the period. From 2000 to 2003, all correlations are positive, meaning that a negative shock on stock returns or SSR changes in both currency areas is generally associated with a euro appreciation. Similarly, $\{\rho_{SC}, \rho_{SC^*}\}$ are negative on average from 2009 to 2013. Therefore, negative shocks on stock returns in both currency areas are mainly associated with US dollar appreciation during this period. This specific exchange rate behavior from 2009 to 2013 can be explained by the safe haven status of the US dollar during financial distresses. Finally, 2008 is the only year where both $\rho_{SL^*}$ and $\rho_{SC^*}$ are positive while $\rho_{SL}$ and $\rho_{SC}$ are negative, suggesting that positive shocks on one currency area is associated with an appreciation of its currency.

\[
\{ \text{Insert Figure} 4 \text{ here } \}
\]

3.3 International diversification from 2000 to 2015

To close our estimation of efficient diversification, the leverage ratio is defined based on the Basel III framework. A minimum leverage ratio ($E/A$) is fixed at 3%, implying a leverage of 32.33. The yearly average conditional variances and correlations reported
in table 2 and 3 are added to the simulation to identify efficient portfolios. Figure 5a reports efficient international diversification on both sides of bank’s balance sheet.

From 2001 to 2003, $\lambda > \psi$, translating the benefit from risk reduction in equity related to correlations with foreign exchange rate $\{\rho_{SL}, \rho_{SL^*}, \rho_{SC}, \rho_{SC^*}\}$. As all correlations are positive, a positive shock on asset returns and financing costs is associated with a US dollar appreciation. When only assets are diversified, equity return volatility increases with the foreign exchange channel. When liability diversification is introduced such as $\psi = \lambda$, foreign exchange movements on the asset side are partly compensated by foreign exchange movements on liabilities because of leverage. An efficient currency mismatch where $\lambda > \psi$ then compensates foreign exchange movements and reduces equity volatility. However, the currency mismatch is relatively limited as $\rho_{L^*C^*} > \rho_{LC}$ and $\sigma_{C^*}^2 > \sigma_C^2$ over this period.

Unsurprisingly, 2008 is characterized by large degree of volatility in all components of the balance sheet: the volatility of stock returns in both currency areas reaches its highest level over the period, while the volatility of the foreign exchange rate and the EA SSR are at their second largest level over the period. 2008 is also the only year where the US asset return volatility is larger than the volatility of EA asset returns. Therefore, the efficient diversification of assets $\psi$ reaches its lowest level in 2008. Despite of the financial crisis, the model still predicts a large international diversification in both sides of the balance sheet as reported in figure 5a, the model predict that almost half of the bank’s balance sheet is in foreign currency. As mentioned previously, $\rho_{SL^*}$ is positive and $\rho_{SL}$ is negative but they share similar magnitudes. Therefore, an equally diversified liability cancels out impact of correlations. Similarly, $\rho_{SC}$ is negative and $\rho_{SC^*}$ is positive, and their magnitude are alike. A diversification of asset close to 0.5 is also justified. Finally, the large values of $\rho_{L^*C^*}$ and $\rho_{LC}$ support the currency match.

{ Insert Figure 5 here }
between assets and liabilities in order to absorb shocks from both sides of the balance sheet, foreign exchange rate risk aside. International diversification is thus compatible with large episodes of financial distress.

From 2009 to 2012, the model in figure 5a predicts a large decline in liability diversification with significant currency mismatch where $\lambda \leq 0.1$ and $\psi \geq 0.6$. During this period, several episodes of financial distress are observed in both the US and the EA financial markets (figure 2). Despite the large volatility in both financial markets, the US stock index still offers lower volatility than its EA counterparts, promoting large $\psi$. Additionally, the large volatility of foreign exchange rate observed during this period does not prevent from a currency mismatch because of the negative and strong correlation $\rho_{SC^\ast}$. Positive shocks on US asset returns are associated with euro appreciation while negative shocks are associated with US dollar appreciation: foreign exchange rate shocks and shocks on US asset returns balance one another. This stabilizing effect of $\rho_{SC^\ast}$ on equity volatility is maximized for relatively low value of efficient $\lambda$, promoting then an efficient currency mismatch. Finally, the US liability is not the best instrument to hedge against shocks on asset returns, foreign exchange rate fluctuations aside. As $\rho_{LC^\ast} > \rho_{L^\ast C^\ast}$, banks in the EA benefit from using domestic liability to cover foreign assets. Interestingly, this period of large and efficient currency mismatch where $\lambda \mapsto 0$ corresponds to the period where European banks faced difficulties to fund themselves in US dollar, especially in 2011.\textsuperscript{13}

After 2012, $\rho_{SC^\ast}$ increases and becomes positive, and the foreign liability turns back to be the best instrument to hedge against shocks on assets, foreign exchange rate aside. The previous currency mismatch where $\psi > \lambda$ is no more efficient. Additionally, it is more efficient to use $L^\ast$ to hedge shocks on both $C^\ast$ and $C$ than $L$ (i.e $\rho_{L^\ast C} > \rho_{LC}$),

\textsuperscript{13}See Ivashina et al. [2015] for more details on this specific period.
explaining that $\lambda > \psi$ in 2014 and 2015.

Figure [5b] reports the observed international diversification of Monetary Financial Institutions (MFIs) in the EA between 2000 and 2015. International diversification of assets, $\psi$, is the ratio of the sum of loans and debt securities denominated in US dollar with a non-EA counterparty held by MFIs to the sum of total loans and debt securities held by MFIs. Alternatively, international liability diversification, $\lambda$, is the ratio of the sum of deposits and debt securities denominated in US dollar with a non-EA counterparty issued by MFIs to the sum of total deposits and debt securities issued by MFIs. Although the magnitude of observed $\psi$ and $\lambda$ are on average 10 times lower than efficient $\psi$ and $\lambda$ reported in figure [5a], three main observations are worth saying. First, both observed and efficient international liability diversification, $\lambda$, share similar trends over the period. After a sharp increase at the beginning of the period, there is an overall reduction in $\lambda$ from 2003 to 2012. In 2012, after the US dollar shortage episode, both the efficient and the observed $\lambda$ start to expand again until 2015. Second, international liability diversification - either observed or efficient one - shows larger fluctuations than international asset diversification. Third, the period going from 2004 to 2008 goes hand in hand with the smallest level of observed and efficient currency mismatch.

Over the period, international diversification on both total assets and liabilities allows an improvement in bank’s resilience, even during financial crisis such as 2008. The model also explains the appearance of currency mismatch and highlight the fact that risks can complement one another. Although magnitudes differ between efficient and observed international diversification, this exercise brings to light some common patterns between what the theoretical model predicts and what the EA banks actually do. All-in-all it suggests that EA banks may follow international portfolio strategies to maximize their

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\[\text{Data are from the ECB MFIs statistics and the BIS Locational Banking Statistics. The European System of Central Banks in the Euro Area are excluded.}\]
resilience. Decomposing completely equity volatility according to each component is then necessary to understand all the consequences of banks’ international position.

4 Conclusion

The financial crisis of 2008 has drawn special attention to international activities of banks and to the international integration of financial markets. Especially, banks are globally exposed to risk on both sides of their balance sheet, including shock on financial assets, funding costs and foreign exchange rate. Therefore, this paper provides an innovative theoretical framework to consider international integration and the impact of international diversification on the resilience of banks.

The model identifies three channels through which international integration and international diversification affects the resilience of banks: the global financial cycle; the within balance sheet channel between assets and liabilities; and the foreign exchange channel. Using data covering the US and the euro area, the paper provides an interesting documentation of the dynamic of the international integration from 2000 to 2015. Considering the characteristics of the international integration, the model predicts that international diversification improves the resilience of banks even during periods of strong international integration such as 2008. Comparing efficient diversification with the international diversification of banks located in the euro are, this papers brings to light similar patterns. All-in-all, it suggests that euro area banks follow strategies which maximize their resilience when they consider their external positions.

Finally, the framework of the model enables intuitive interpretations of the results and it introduces many potential extensions. This paper focuses on the balance sheet diversification regarding different geographic locations. However, one may consider other
types of diversification, including business diversification. Depending on the research questions, many hypotheses can be adapted.
Appendix

Descriptive statistics:

Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skew</th>
<th>Kurt.</th>
<th>ADF</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX</td>
<td>-0.00001</td>
<td>0.00663</td>
<td>-.02090</td>
<td>4.42408</td>
<td>-1.422</td>
<td>3934</td>
</tr>
<tr>
<td>US SSR</td>
<td>-0.00001</td>
<td>.00019</td>
<td>-.67017</td>
<td>17.11876</td>
<td>-0.410</td>
<td>3934</td>
</tr>
<tr>
<td>EA SSR</td>
<td>-0.00001</td>
<td>.00019</td>
<td>.98390</td>
<td>19.19070</td>
<td>-0.791</td>
<td>3934</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.0001</td>
<td>0.01279</td>
<td>-.18733</td>
<td>10.82903</td>
<td>-0.879</td>
<td>3934</td>
</tr>
<tr>
<td>Euro Stoxx 50</td>
<td>-0.00009</td>
<td>0.01551</td>
<td>.01291</td>
<td>7.14403</td>
<td>-2.231</td>
<td>3934</td>
</tr>
</tbody>
</table>

Note: summary statistics are calculated using the first difference of log-level data (except for SSR which are initially in level instead of log-level) and unit root tests concern level data. ADF lags chosen by AIC.
(a) International stock market indices
(b) Shadow short rates (SSR), daily returns
(c) Exchange rate

**Figure 1: Financial markets:**
Sources: Bloomberg, The Reserve Bank of New Zealand.
DCC GARCH and conditional correlations

For each bivariate DDC GARCH, the estimation goes into two steps. First, it estimates the conditional volatility of each one of the two series \( \{i, j\} \) from univariate GARCH(1,1). Then, the bivariate DCC GARCH captures from the first step the dynamic correlation between the two series. Suppose \( r_t \) a 2x1 vector of returns of 2 assets at time \( t \), \( H_t \) a 2x2 matrix of conditional variances of \( r_t \) at time \( t \) and \( z_t \) a 2x1 vector of iid errors such that \( E[z_t] = 0 \) and \( E[z_t z_T^T] = I \). Then, univariate GARCH is such that:

\[
    r_t = H_t^{1/2} z_t
\]  

Decomposing the covariance matrix \( H_t \) into conditional standard deviation \( D_t \) from univariate GARCH, and a correlation matrix \( R_t \) capturing the dynamic correlation \( \{i, j\} \), the DCC GARCH introduces the following extension:

\[
    H_t = D_t R_t D_t
\]  

Where the varying conditional correlation matrix \( R_t \) is defined as:

\[
    R_t = (I \odot Q_t)^{-1/2} Q_t (I \odot Q_t)^{-1/2}
\]

\[
    Q_t = (1 - a - b) \tilde{Q} + a \epsilon_{t-1} \epsilon_{t-1}^T + b Q_{t-1}
\]

Therefore, the dynamic matrix process \( Q_t \) is a function of \( \tilde{Q} \), the unconditional correlation matrix of the standardized errors \( \epsilon_t \). Our results suggest that all correlations are mean-reverting process where \( (a + b) < 1 \). Additionally, all Wald tests reject the null hypothesis where \( a = b = 0 \): conditional correlations are dynamic.
Table 2: Conditional variances. $S$, $C$, $C^*$, $L$ and $L^*$ refer to the exchange rate, the eurostoxx 50 index, the S&P500 index, the euro Shadow Short Rate and the US Shadow Short Rate respectively.
Figure 2: Conditional variances from DCC GARCH(1,1)
Figure 3: Assets and liabilities: conditional correlations from bivariate DCC GARCH (1,1) estimations
Figure 4: Exchange rate: conditional correlations from bivariate DCC GARCH (1,1) estimations
Table 3: Conditional correlations. $S$, $C$, $C^*$, $L$ and $L^*$ refer to the exchange rate, the eurostoxx 50 index, the S&P500 index, the euro Shadow Short Rate and the US Shadow Short Rate respectively.
Efficient international diversification

Figure 5: International diversification of banks: International diversification of assets and liabilities - $\psi$ and $\lambda$, respectively - measures the share of assets or liabilities denominated in US dollar with a foreign counterparty. Efficient international diversification is such that the volatility of bank’s equity is minimized according to the theoretical model. Observed international diversification measures the international diversification of MFIs in EA (source: ECB data, BIS LBS statistics, own calculations).
References


