Long memory and power law in coherency between realized volatility and trading volume

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Abstract

The nature of the relationship between trading volume and volatility series has been widely studied from a short-run or a long-run perspective, but overall the literature provides mixed results. We investigate this issue for the thirty components of the Dow Jones stock market index in the light of a recent general p-component model who can be related to the theoretical financial literature on the transmission mechanisms of the information flow. Detecting power law in coherency at frequencies beyond zero, our analysis shows that trading volume and volatility are linked through a persistent common factor dwarfed by more persistent idiosyncratic components in the case of more than half of the firms under analysis. In contrast to cointegration theory, this phenomenon is compatible with both the mixture of distributions hypothesis and the sequential arrival of information hypothesis. A subsequent non-parametric phase spectrum analysis reveals that in all cases the former hypothesis is retained.

Keywords: p-component model, realized volatility, trading volume, mixture of distributions hypothesis, sequential arrival of information
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1. Introduction

The nature of the relationship between trading volume and volatility in financial markets is a highly debated issue. One strand of the literature argues in favor of a joint dependence of the two variables upon a common unobservable arrival of information process. This mixture of distributions hypothesis (MDH hereafter) is theoretically discussed in Clark (1973), Tauchen and Pitts (1983), Andersen (1996) and Liesenfeld (2001). Regarding empirical evidence in favor of the MDH, one can mention among others Bollerslev and Jubinski (1999), Luu and Martens (2003), Ané and Ureche-Rangau (2008), Park (2010), Jawadi and Ureche-Rangau (2013) and Rossi and Santucci de Magistris (2013a).[1] Interestingly, most of these studies account for the persistent nature of trading volume and volatility series, and thus investigate the possibility of long-run dependence. As discussed in Bollerslev and Jubinski (1999), the rational for this long-run version of the MDH is the possibility of heterogeneous responses to news in the short run. The authors test this modified version of the MDH by simply investigating whether the long memory behavior of the two time series is similar. Rather than using GARCH-type models, they approximate volatility by the daily squared and absolute returns, but unfortunately, those measures are very noisy. More importantly, they do not formally test for the presence of common long-run dependence (fractional cointegration), although they mention this possibility as an interesting avenue for future research.

Ané and Ureche-Rangau (2008) go further in the long memory analysis and show that trading volume series are uni-fractal processes whereas volatility series are multi-fractal processes. They argue that this divergent scaling structure contradicts the MDH in the long run but that commonalities might exist in the short run. Indeed, as argued earlier by Liesenfeld (2001), accounting for the long memory behavior of the series is important but the possibility of short-run dependence should not be neglected either. Using stochastic volatility and GARCH-type models, the author finds that volatility is more likely to be driven by the information arrival process in the short run and by the sensitivity to the new information in the long run. Conversely, trading volume is essentially driven by the information arrival process whereas the sensitivity to the new information is irrelevant. Luu and Martens (2003) estimate both a GARCH-type model extended to the de-trended volume and a bivariate VAR model including volume and realized volatility variables. The authors stress the importance of using realized volatility measures and finally conclude in favor of a bi-directional causality, thereby supporting the MDH. Park (2010) uses a GARCH-type approach too, but considers a modified version of the MDH designed to incorporate both positive and negative surprising information. His results are supportive of the modified MDH. Rossi and Santucci de Magistris (2013a) pursue the analysis of Bollerslev and Jubinski (1999) and test for the presence of fractional cointegration when volatility is proxied by the realized variance. They find no evidence of

[1] See also Harris (1986; 1987), Lamoureux and Lastrapes (1990) for earlier studies.

long-run dependencies. On the contrary, they find strong evidence of short-run non-Gaussian extreme dependence by using a fractionally integrated VAR copula-based model.

A second strand of the literature argues that the information signal is disseminated randomly and sequentially to market participants. Accordingly, market equilibrium is reached only once all traders have received the information and adjusted their positions. This sequential arrival of information (SAI) theory has been developed by Copeland (1976), Jennings et al. (1981) and Smirlock and Starks (1988). Because the SAI hypothesis implies the existence of incomplete equilibria during the dissemination process, it has strong implications in terms of market efficiency. The SAI hypothesis has also found a lot of empirical support in the literature (see e.g. Lobato and Velasco 2000, Nielsen 2009, Berger et al. 2009, Fleming and Kirby 2011, Mougoué and Aggarwal 2011, Tseng et al. 2015).[2] Some of these studies do not test directly for the SAI but the way they reject the MDH argues in favor of the SAI. For instance, Fleming and Kirby (2011) investigate whether the series have a common long memory behavior when relying on realized volatility measures. Conversely to Bollerslev and Jubinski (1999), they conclude against this hypothesis and thus against the MDH. Berger et al. (2009) reformulate volatility as a combination of information flow and market sensitivity to this information. Using fractional cointegration techniques, they study the relationship between trading volume and market sensitivity and conclude against the MDH in the long-run. In a very recent paper, Tseng et al. (2015) deal directly with the SAI hypothesis on Exchange Traded Fund assets (ETF). They account for the persistent nature of ETF volatility by means of heterogeneous auto-regressive models and reveal that trading volume contains useful information to improve volatility prediction.

The MDH as well as the SAI hypotheses imply a positive relationship between trading volume and volatility. Interestingly, some articles provide results that justify a negative relationship between the two variables. For instance, Li and Wu (2006) adopt a microstructure approach and generalize Andersen (1996)'s model by decomposing trading volume into "informed" and "liquidity" components. They find that informed trading volume and volatility are positively correlated whereas liquidity volume and volatility are negatively correlated and thus conclude in favor of the generalized MDH. Giot et al. (2010) stress the importance of "good" and "bad" volatility, respectively, that are related to the continuous and the discontinuous jump components of volatility. They find a positive relationship between trading volume and "good" volatility and a negative relationship between trading volume and "bad" volatility and hence conclude in favor of the MDH too. Finally, Mougoué and Aggarwal (2011) focus essentially on the possible nonlinear nature of the relationship between trading volume and volatility. They conclude in favor of a bidirectional nonlinear Granger causality between the series but with a negative sign, thereby concluding against the MDH.

[2]See also Richardson and Smith (1994), Lamoureux and Lastrapes (1994), Hiemstra and Jones (1994) for earlier studies.

A particularly relevant study for our analysis is that of Lobato and Velasco (2000) who address the MDH in a very interesting way. In a first step they analyze the long memory behavior of each time series. In a second step, they go further relatively to Bollerslev and Jubinski (1999) and Fleming and Kirby (2011) by investigating whether the squared coherency between the two series is 1 at zero frequency, i.e. in the "very long-run". Indeed, as demonstrated by Levy (2002) and Nielsen (2004), (fractional) cointegration theory implies a unit squared coherency at zero frequency. In an empirical illustration on the 30 components of the Dow Jones index they find all squared coherencies below 0.4 and conclude against the presence of fractional cointegration and thus against MDH.

In this paper we draw on the works of Bollerslev and Jubinski (1999) and Lobato and Velasco (2000) to propose a new frequency domain analysis of the validity of MDH in the case of the thirty components of the Dow Jones stock market. Our approach relies on a p-component model as the one introduced by Sela and Hurvich (2012) to define the anti-cointegration phenomenon, i.e. the presence of a less persistent common factor that would be dwarfed by the more persistent individual components of two time series. The main advantage of this framework is that it can generate a wide range of behaviors including balanced cointegration, unbalanced cointegration, anti-cointegration and the absence of any type of commonalities near zero frequency.

The originality of our paper consists in adopting this very flexible approach to investigate the possibility of observing a form of dependence of trading volume and volatility anywhere in between the short-run and the long-run and not exclusively at the origin of the spectrum, i.e. with a focus on the long-run MDH, as do the previous works cited above. Indeed, by investigating the possibility of observing power law coherency between trading volume and realized volatility at frequencies near zero but different from it, one can unravel the presence and persistence level of the common and the idiosyncratic factors driving each variable. A subsequent non-parametric phase analysis at near zero frequencies characterized by high squared coherency is used to detect whether significant lead-lag effects occur between the two variables, which would support the SAIH.

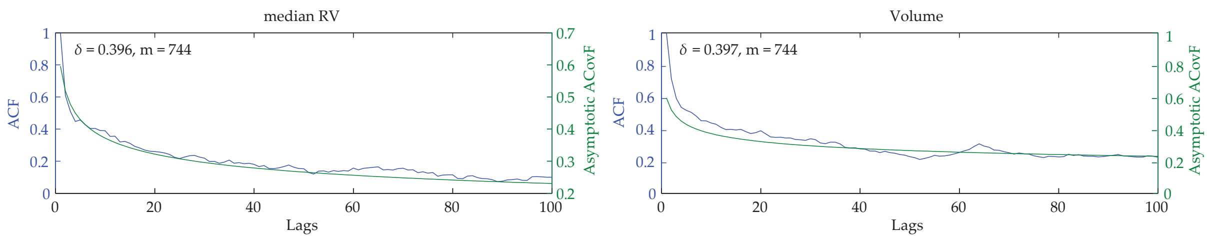
In the empirical illustration, we find strong evidence that trading volume and volatility are composed of two persistent components for more than half of the firms under analysis. The first component is very persistent and idiosyncratic while the second one is less persistent, common to both series, but dwarfed by the first component. The presence of this persistent hidden common factor is consistent with the theoretical financial literature on the transmission mechanisms of the information flow to the two variables. At the same time, the long-run phase analysis indicates that trading volume and volatility exhibit positive contemporaneous dependence, which provides evidence in favor of the MDH at low but non-zero frequencies. This new result may explain the mixed findings of the literature and notably why neither fractional cointegration, i.e. long-run analyses, nor short-run techniques are able to clearly detect the common factor. A series of robustness checks support our main findings.

The rest of the paper is organized as follows. In Section 2 we present the econometric techniques used. Section 3 describes the realized measures of volatility. Sections 4 and 5 detail the empirical results and Section 6 concludes.

2. Methodology

Numerous studies document the persistent nature of volatility and support the fact that realized measures exhibit long memory behavior. Using Apple as an example, Figure 1 plots the asymptotic approximation of autocovariances proposed by Lieberman and Phillips (2008), and the autocorrelation functions of the median realized variance measure and of the trading volume.[3] The two series present clear signs of slow rate of decay and thus exhibit direct evidence of the presence of long memory. Unreported results show that the same holds true for the volatility and the volume of all the other firms in the sample.

Figure 1: Asymptotic autocovariance and autocorrelation functions for Apple.



Notes: The autocovariance is derived from the asymptotic approximation proposed by Lieberman and Phillips (2008). It uses the long memory estimator of Robinson (1995) with a bandwidth parameter selected such that the autocovariance tracks the autocorrelogram.

These stylized facts support the need to perform a long run analysis of the relationship between trading volume and volatility. But, in contrast to the existing literature that tests only for the presence of cointegration, we rely on a more general framework that allows for a large variety of long run behaviours. Hereafter, we denote by x_{1t} and x_{2t} the trading volume and the volatility variables, respectively. In the rest of the section, we detail the model, distinguish between the underlying common and idiosyncratic factors and discuss parameter estimation.

2.1. The p-component model

As demonstrated in Levy (2002) and Nielsen (2004), frequency domain bivariate (fractional) cointegration systems imply a unit squared coherency and a phase shift equal to 0 at zero frequency. In

[3]This approximation requires to pre-estimate semi-parametrically the long memory parameter (δ) and to choose a bandwidth parameter (m) such that the autocovariance tracks perfectly the autocorrelogram. For the former, the long memory estimator of Robinson (1995) is used. In the top left corner of each subplot we report m and $\hat{\delta}$.

contrast, when there is no cointegration, the most persistent components, i.e those dominating the long-run dynamics of each time series, are idiosyncratic. This implies a power law in autospectra but not in the squared coherency at zero frequency. However, as discussed in Sela and Hurvich (2012), this result does not exclude the presence of a less persistent common factor that would be dwarfed by the more persistent individual components. This phenomenon, that the authors named anti-cointegration, is likely to appear when one generalizes the traditional bivariate model where $x_t = (x_{1,t}, x_{2,t})'$ is driven by a bivariate sequence of innovations, to the more realistic case where a p-variate sequence of innovations drives x_t,

$$x_t = \sum_{l=-\infty}^{\infty} \psi_l (\varepsilon_{1,t-l}, \dots, \varepsilon_{k,t-l}, \dots, \varepsilon_{p,t-l})', \tag{1}$$

where ψ_l is a $2 \times p$ real-valued matrix of infinite moving average coefficients, $\varepsilon_{t-l} = (\varepsilon_{1,t-l}, \dots, \varepsilon_{p,t-l})'$ is a p-variate *i.i.d.* white-noise process with finite fourth moment and $Cov(\varepsilon_t) = 2\pi\Sigma$ is a symmetric and positive definite matrix. In this p-component model, under the stationarity assumption, the spectral density of x_t is defined as

$$f(\lambda) = \Psi(\lambda)\Sigma\Psi(\lambda)^*, \quad \lambda \in [-\pi, \pi]. \tag{2}$$

Under Assumptions 2-4 of Sela and Hurvich (2012), the transfer function $\Psi(\lambda) = \sum_{l=-\infty}^{\infty} \psi_l e^{-i\lambda l}$, defined for $\lambda \in [-\pi, \pi]$, is a linear filter that can be decomposed as $\Psi_{j,k}(\lambda) = \left(1 - e^{-i\lambda}\right)^{-\tilde{\delta}_{j,k}} \tau_{j,k}(\lambda) e^{-i\varphi_{j,k}(\lambda)}$, where the long-run, the short-run, and the phase shift properties of the k^{th} innovation of $x_{j,t}$, $j = 1, 2$, are given by $\left(1 - e^{-i\lambda}\right)^{-\tilde{\delta}_{j,k}}$, $\tau_{j,k}(\lambda)$, and $\varphi_{j,k}(\lambda)$, respectively. In the long-run, $x_{j,t}$ inherits the long memory behaviour of the k^{th} component having the largest order of integration and satisfying $\tau_{j,k}(0) > 0$,

$$\delta_j = \max_{k:\tau_{j,k}(0)>0} (\tilde{\delta}_{j,k}). \tag{3}$$

Hence, near the origin, i.e. $\lambda \to 0^+$, the auto-spectrum of $x_{j,t}$ is given by $f_j(\lambda) \sim G_j \lambda^{-2\delta_j}$, with

$$\begin{aligned} G_j &= \lim_{\lambda \to 0^+} \sum_{k=1}^{p} \sum_{l=1}^{p} \sigma_{kl} \tau_{j,k}(\lambda) \tau_{j,l}(\lambda) e^{i(\varphi_{j,k}(\lambda) - \varphi_{j,l}(\lambda))} \mathbb{1}(\tilde{\delta}_{j,k} = \delta_j) \mathbb{1}(\tilde{\delta}_{j,l} = \delta_j) \\ &= \lim_{\lambda \to 0^+} \sum_{k=1}^{p} \sum_{l=1}^{p} \tilde{G}_{kl}(\lambda) \mathbb{1}(\tilde{\delta}_{j,k} = \delta_j) \mathbb{1}(\tilde{\delta}_{j,l} = \delta_j), \end{aligned} \tag{4}$$

given that $(1 - e^{-i\lambda})^{-\tilde{\delta}_{jk}} = \lambda^{-\tilde{\delta}_{jk}} e^{-i(\pi-\lambda)\tilde{\delta}_{j,k}/2} (1 + O(\lambda^2))$ and that the imaginary part annihilates in the autospectra, and where $\mathbb{1}(.)$ denotes the indicator function. Using $(1 - e^{-i\lambda})^{-\tilde{\delta}_{jk}} = \left|2 \sin \frac{\lambda}{2}\right|^{-\tilde{\delta}_{jk}} e^{-i(\pi-\lambda)\tilde{\delta}_{j,k}/2}$

to rewrite $\Psi_{j,k}(\lambda)$ at all frequencies, the cross-spectral density takes the form of

$$
\begin{aligned}
f_{12} &= \sum_{k=1}^{p}\sum_{l=1}^{p}\left|2\sin\frac{\lambda}{2}\right|^{-\tilde{\delta}_{1k}-\tilde{\delta}_{2l}}\sigma_{kl}\tau_{1,k}(\lambda)\tau_{2,l}(\lambda)e^{i(\varphi_{1,k}(\lambda)-\varphi_{2,l}(\lambda)-(\pi-\lambda)\tilde{\delta}_{1,k}/2+(\pi-\lambda)\tilde{\delta}_{2,l}/2)} \\
&= \sum_{\tilde{q}=1}^{\tilde{Q}}\left|2\sin\frac{\lambda}{2}\right|^{-2\delta_{12}(\tilde{q})}e^{-i(\pi-\lambda)\bar{\delta}_{12}(\tilde{q})}\tilde{G}(\lambda;\tilde{q}) \qquad (5)
\end{aligned}
$$

where $\tilde{q}=1,\dots,\tilde{Q}$ is associated with the set $S_{\tilde{q}}$, which regroups all couples $\{(k,l):k,l\in\{1,\dots,p\}\}$ for which $\tilde{\delta}_{1,k}+\tilde{\delta}_{2,l}$ is constant. The sets are defined such that $\delta_{12}(\tilde{q})=(\tilde{\delta}_{1,k}+\tilde{\delta}_{2,l})/2$ fulfils $\delta_{12}(\tilde{q})>\delta_{12}(\tilde{q}+1)$, i.e. $\delta_{12}(1)=(\delta_1+\delta_2)/2$. For a given $\delta_{12}(\tilde{q})$ we also define $\bar{\delta}_{12}(\tilde{q})=(\tilde{\delta}_{1,k}-\tilde{\delta}_{2,l})/2$. Under Assumption 5 of Sela and Hurvich (2012), in the vicinity of the origin the cross-spectral density is driven by its first term rather than $\tilde{G}(\lambda;\tilde{q})$. Using $\left|2\sin\frac{\lambda}{2}\right|^{-\tilde{\delta}_{jk}}e^{-i(\pi-\lambda)\tilde{\delta}_{j,k}/2}\sim\lambda^{-\tilde{\delta}_{jk}}e^{-i(\pi-\lambda)\tilde{\delta}_{j,k}/2}$ and Assumptions 2-4 of Sela and Hurvich (2012), as $\lambda\to 0^+$, the cross spectrum has a power law behaviour

$$
f_{12}(\lambda)\sim G_{12}\lambda^{-2\delta_{12}}, \qquad (6)
$$

the coherency satisfies

$$
\rho(\lambda)\sim\frac{G_{12}}{\sqrt{G_1G_2}}\lambda^{-2\delta_\rho}=\frac{|G_{12}|}{\sqrt{G_1G_2}}\lambda^{-2\delta_\rho}e^{i\varphi(\lambda)}, \qquad (7)
$$

with $\delta_\rho=\delta_{12}-(\delta_1+\delta_2)/2\le 0$ as δ_{12} is bounded from above by $\delta_{12}(1)$ and

$$
\varphi(\lambda)=\arg\left(\sum_{\tilde{q}=1}^{\tilde{Q}}\left|2\sin\frac{\lambda}{2}\right|^{-2(\delta_{12}(\tilde{q})-\delta_{12})}e^{-i(\pi-\lambda)\bar{\delta}_{12}(\tilde{q})}\tilde{G}(\lambda;\tilde{q})\right) \qquad (8)
$$

summarizes the possibly complex long run phase behaviour. When $\varphi(\lambda)$ is non-null close to the zero frequency, its derivative $\varphi(\lambda)'$ is of particular interest because it indicates whether x_{1t} leads or lags x_{2t} by $\varphi(\lambda)'$ periods at frequency λ. Near zero frequency, the phase is of form $\varphi(\lambda)=\varphi_0+\varphi_1\lambda^\alpha+o(\lambda^\alpha)$ (see Sela 2010). When $\alpha\ge 1$ and $\varphi(\lambda)'$ goes to 0 at zero frequency, there is no lead-lag effect in the long run. However, in some particular cases $0<\alpha<1$ arises and $\varphi(\lambda)'\to\infty$ as $\lambda\to 0^+$, thereby implying that one series leads the other by increasing amounts at larger lags proportionally to $n^{1-\alpha}$.

At this stage, $f_{12}(\lambda)$ has a very general form. It allows for power law in coherency when $\delta_{12}<(\delta_1+\delta_2)/2$. This phenomenon reflects the presence of a common factor dwarfed by at least one idiosyncratic factor and implies that the spectral coherency goes to 0 at zero frequency. Recall that the cointegration theory implies the opposite, i.e. $\delta_\rho=0$, $\varphi(0)=0$ and $\rho(0)=1$, and this case is also handled by the p-component model. Interestingly, unbalanced cointegration (see Hualde 2006; 2014) can also emerge in this framework if, for example, a common persistent component with long memory $\tilde{\delta}_c$ exists and $\delta_j=\tilde{\delta}_c<\delta_{j'}$

for $j \neq j'$ with $j, j' \in \{1, 2\}$. In such a case, the presence of power law in coherency will depend on the covariance structure of Σ. Typically, if Σ is diagonal, a sufficient condition to observe $\delta_\rho < 0$ is $\tilde{q}_0 > 1$. Note that unbalanced cointegration and anti-cointegration definitions overlap in this particular situation. At the opposite, when $\sigma_{j'c} \neq 0$, $\tilde{q}_0 = 1$ and unbalanced cointegration without power law in coherency arises. This is however different from balanced cointegration as $\rho(0) < 1$ and the phase is non-null. Another well known specification that the p-component model embeds is the standard bivariate causal VARFIMA (i.e. $p = 1$). For this model, the phase shift reduces to the linear form $\varphi(\lambda) = -a\lambda$ for $a \in \mathbb{R}$, $f_{12}(\lambda)$ simplifies to $|G_{12}|\lambda^{-2\delta_{12}} e^{i\pi(\delta_2 - \delta_1)/2}$ as $\lambda \to 0^+$, and no power law coherency occurs, i.e. $\delta_\rho = 0$. To summarize, the p-component model can generate a wide range of long run behaviours including balanced cointegration, unbalanced cointegration, anti-cointegration and the absence of any type of commonalities near zero frequency.

2.2. *Common vs. idiosyncratic factors of volatility and volume*

In this subsection we motivate the suitability of the p-component model to study the relationship between trading volume and volatility. By relying on the results from existing empirical and theoretical works, we grasp insights into which financial factors and mechanisms may be at play. Consistently with Bollerslev and Jubinski (1999), at least one of the innovations' components, $\varepsilon_{k,t}$, $k \in 1, \ldots, p$, should be common and reflect the aggregated information-arrival process. However, conversely to the existing literature, the p-component model relaxes the hypothesis that the information-arrival process is the most persistent one. A natural question that arises is hence how to interpret the presence of idiosyncratic factors.

On the empirical side, e.g., Liesenfeld (2001) shows that the information flow and the sensitivity of the market to that information arrival are relevant factors for the dynamics of volatility, but the latter is irrelevant for the trading volume and cannot be considered as a common factor. Berger et al. (2009) show that both factors are persistent and may explain the time-variation of volatility. On the theoretical side, the presence of persistent idiosyncratic shocks builds on the literature on liquidity. For instance, Darolles et al. (2015) extend the model of Tauchen and Pitts (1983) by introducing liquidity as a leading factor of volume although the price and the volatility processes are free of it, while Darolles et al. (2017) explicitly account for the persistent nature of liquidity. Johnson (2008) develops a microstructure model to jointly analyze trading volume and liquidity and draws a different conclusion from that of Darolles et al. (2015) as he finds that trading volume comoves with liquidity risk rather than liquidity, the latter being in fact tied to volatility. This result suggests that idiosyncratic factors are likely to exist in the two equations of the model and they may exhibit different degrees of persistence, possibly greater or lower than the common factor.

Another strand of the literature is interested in the underlying economic mechanisms through which trading volume and volatility are linked (see e.g. Bollerslev et al 2018). Most of this recent literature develops theoretical foundations for the link between trading volume and price volatility by making use of disagreement-based models that deviate from rational expectations. These models generally assume that traders receive the same common information but differ in the way they interpret it. They may have different expectations, opinions or different likelihood functions when they Bayesian update, which corresponds to different channels of transmission of the information flow to the variables of interest. For example, in Banerjee (2011) investors can update their beliefs via a rational expectations mechanism or they can agree to disagree, which results in persistent differences in opinion. The model of Osambela (2005) distinguishes between the low frequency component of stock return volatility which is (negatively) driven by market liquidity and implicitly by aggregate consumption and disagreement risks and the transitory (high-frequency) volatility factor driven by funding illiquidity risk. Atmaz and Basak (2018) develop a dynamic general equilibrium model that allows to disentangle the effects of belief dispersion, which reflects the extra uncertainty investors face, from those of Bayesian learning on return, volatility, and trading volume. Overall, these theoretical approaches seem to be able to explain empirical observations: i) dispersion in expectations exacerbates volatility and leads to higher trading volume, ii) positive correlation exists between volatility and trading volume.

As the p-component model seems to be an adequate framework to investigate the presence of latent common factors in the dynamics of trading volume and volatility, we now introduce the estimator of the power-law in the cross-spectrum of the variables and that of the phase behaviour, which will allow us to distinguish between MDH or SAIH.

2.3. *Estimation of power law in coherency*

Detecting power law in coherency (anti-cointegration or unbalanced cointegration) requires to estimate $\delta_\rho = \delta_{12} - 0.5(\delta_1 + \delta_2)$, i.e., the long memory parameters in autospectra and the cross-spectrum. Sela and Hurvich (2012) address this issue by proposing a model-free frequency-domain approach.

There is a large literature dealing with the estimation of long memory in autospectra, but only a few studies mention the cross-spectrum case, i.e. δ_{12}. Lobato (1997) discusses the possibility of power law coherency, but does not propose an estimator of δ_ρ. He focuses essentially on the estimation of δ_1 and δ_2 by extending the averaged periodogram estimator (APE) introduced by Robinson (1994) to the multivariate case by considering the averaged periodogram matrix,

$$\hat{F}(\lambda) = \frac{2\pi}{n} \sum_{j=1}^{\lfloor n\lambda/2\pi \rfloor} I(\lambda_j), \text{ with } I(\lambda) = \left(\frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} x_t e^{i\lambda_j t} \right) \left(\frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} x_t e^{i\lambda_j t} \right)^*,$$

and assumes that $\delta_{12} = 0.5(\delta_1 + \delta_2)$. Sela and Hurvich (2012) relax this assumption and extend the APE to estimate the power law in the cross-spectrum. Their estimator of δ_{12} is given by

$$\hat{\delta}_{12} = \frac{1}{2} - \frac{\log\left(|\hat{F}_{kl}(q\lambda_m)|/|\hat{F}_{kl}(\lambda_m)|\right)}{2\log q}, \text{ for } q \in (0,1). \tag{9}$$

In the following, we draw on their paper and set $q = 0.5$. For $\delta_j \in (0, 1/2)$, $j = \{1, 2\}$, this estimator is consistent, has an asymptotic normal distribution if $\delta_j < 1/4$ and has an asymptotic non-normal distribution when $\delta_j > 1/4$. When $\delta_{12} \in (0, 1/2)$, the estimator remains consistent, but Sela and Hurvich (2012) investigate the limit distribution only for $\delta_{12} < 1/4$. Accordingly, one is not able to interpret the standard errors when $\delta_{12} > 1/4$. When $\delta_j \in [1/2, 1)$, no asymptotic result exists although one can conjecture that the maximal memory estimator of Robinson and Marinucci (2000) can be extended to the framework of Sela and Hurvich (2012). A preliminary estimation of long memory in volatility and volume reveals the presence of moderate non-stationarity. Consequently, we rely on the local polynomial Whittle (LPW) estimator of Frederiksen et al. (2012) to estimate δ_j, as it accommodates both nonstationarity and the presence of long-run noises in the auto-spectra. In contrast, our estimates of δ_{12} are mostly found to be confined in $(0, 1/2)$ and sometimes in $(1/4, 1/2)$. This results in a lack of uniform convergence that considerably complexifies the analysis of $\hat{\delta}_\rho$. To solve this issue and carry out inference on the power law coefficient, we adapt the bootstrap procedure of Arteche and Orbe (2016) to the cross-spectrum averaged periodogram estimator. This approach consists in standardizing the periodogram locally by dividing it by an expression that is proportional to the spectral density function around the origin prior to resampling. The percentile bootstrap confidence interval is then computed for each $\hat{\delta}_\rho$.

2.4. Phase spectrum estimation

Unfortunately, the general approach of Sela and Hurvich (2012) neither distinguishes between anti-cointegration and unbalanced cointegration, nor identifies the structure of the p-component model and cannot reveal the phase behaviour. In other words, it allows to detect the presence of hidden common factors but cannot conclude in favour of one of the two hypotheses, MDH or SAIH. Recall that the SAIH predicts that trading volume should lead volatility. Conversely, the absence of lead-lag effects should reveal a bi-directional feedback between the series, thereby supporting the MDH. In our frequency domain analysis, a convenient approach to ascertain which variable leads the system is to compute non-parametrically the phase between x_{1t} and x_{2t}. It is well-known that the cross-spectrum, $f_{12}(\lambda)$, is a complex-valued function admitting the decomposition $f_{12}(\lambda) = c_{12}(\lambda) - iq_{12}(\lambda)$, where $c_{12}(\lambda) = \text{Re}[f_{12}(\lambda)]$ is the co-spectrum of x_t and $q_{12}(\lambda) = -\text{Im}[f_{12}(\lambda)]$ is the quadrature spectrum of x_t. It follows that the phase difference, defined as

$$\varphi(\lambda) = \arg\left(c_{12}(\lambda) - iq_{12}(\lambda)\right), \tag{10}$$

measures the lead or lag effect of x_{1t} over x_{2t} at frequency λ.[4] Recall that x_{1t} is the trading volume and x_{2t} stands for the volatility. If $\varphi(\lambda) = 0$, x_{1t} and x_{2t} move in phase (together) at frequency λ, which would confirm the MDH. If $\varphi(\lambda) \in (-\pi/2, 0)$, the two series move in phase with a pro-cyclical effect coming from the fact that the volatility (x_{2t}) leads the trading volume (x_{1t}). Similarly, when $\varphi(\lambda) \in (0, \pi/2)$, the series move in phase but with a pro-cyclical effect coming from the fact that trading volume leads volatility. This case should confirm the SAI hypothesis. Conversely, if $\varphi(\lambda) \in (-\pi, -\pi/2)$ or $\varphi(\lambda) \in (\pi/2, \pi)$, both series experiment an anti-phase movement (counter-cyclical) and either x_{1t} leads x_{2t}, or x_{2t} leads x_{1t}, respectively. This result would be in line with Giot et al. (2010) and Mougoué and Aggarwal (2011) because it would describe a negative relationship between trading volume and volatility.

3. Volatility proxies

To construct precise ex-post proxies of volatility, econometricians consider either the model-free approach, or the reduced-form, i.e. model-based approach. In the following, to reduce misspecification risk, we adopt the first approach, which consists mainly in using high-frequency data to compute ex-post realized measures of volatility at a lower frequency. These realized measures of volatility are generally more informative about the current level of volatility than the squared returns. Besides, the more recent proxies also take into account the existence of jumps and market microstructure noise that are likely to pollute very high-frequency data.

In our analysis, we first consider the so-called realized variance estimator (*RV* hereafter) discussed in Andersen and Bollerslev (1997). The RV estimator is obtained by choosing a sampling frequency Δ (set to 5 minutes in our study) and by summing the $M = 1/\Delta$ squared intraday returns over a day t so that

$$RV_t(\Delta) = \sum_{j=1}^{M} r_{t,j}^2,$$

where $r_{t,j} = p_{t,j\Delta} - p_{t,(j-1)\Delta}$, with p_t the logarithmic asset price. Assuming a continuous stochastic volatility diffusion model for the price process and by the theory of quadratic variation, $RV_t(\Delta)$ converges to the so-called integrated variance as $M \to \infty$, i.e. $RV_t(\Delta) \xrightarrow{p} \int_0^t \sigma_s^2 ds$. Nonetheless, there is a large consensus in the literature in favor of a jump-contaminated price dynamics implying that

$$RV_t(\Delta) \xrightarrow{p} \int_{t-1}^{t} \sigma^2(\tau) d\tau + \sum_{j=1}^{N_t} \kappa_{t,j}^2,$$

where $\kappa_{j,t}$ is the size of the jump j on day t, and N_t is the number of jumps on that day. Regarding the MDH, it is interesting to investigate whether the non-continuous part of the quadratic variation is likely

[4] The phase difference can be equivalently expressed in time units (time shift of x_{1t} over x_{2t} at frequency λ) by $\tau(\lambda) = \varphi(\lambda)/\lambda$.

to impact the results. To disentangle the discrete and the continuous components, we first consider the so-called bi-power variation (BPV) measure introduced by Barndorff-Nielsen and Shephard (2004):

$$BPV_t(\Delta) = \frac{\pi}{2}\frac{M}{M-1}\sum_{j=1}^{M-1}|r_{t,j}||r_{t,j+1}|.$$

As the frequency increases, BPV converges to the integrated variance, but in practice not all jumps are eliminated. We hence consider also the jump-robust measure introduced by Andersen et al. (2012), named median realized variance ($medRV$ hereafter), and defined as follows

$$medRV_t(\Delta) = \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{M}{M-2}\right)\sum_{j=2}^{M-1}\text{med}(\mid r_{t,j-1}\mid,\mid r_{t,j}\mid,\mid r_{t,j+1}\mid)^2.$$

Compared to the BPV estimator, the $medRV$ is designed so that the impact of jumps vanishes completely except in the case of two consecutive jumps (which is extremely rare at the sampling frequencies used in empirical applications).

Another issue that is likely to affect our results is the presence of microstructure frictions, such as bid-ask bounce or infrequent trading, when the sampling frequency is high. The realized kernel (RK) estimator of Barndorff-Nielsen et al. (2008) is specifically designed to account for this stylized fact

$$RK_t(\Delta) = \sum_{h=-H}^{H} k(h/(H+1))\gamma_{h,t},$$

where H is a bandwidth determined by following the recommendations of Barndorff-Nielsen et al. (2009), $\gamma_{h,t} = \sum_{j=|h|+1}^{M} r_{t,j}r_{t,j-|h|}$ and $k(.)$ is the Parzen kernel function. For robustness reasons, all these realized measures are used in the sequel of the paper.

4. Empirical results

4.1. Data

We revisit the question of the nature of the relationship between trading volume and volatility by focusing on the Dow Jones Industrial Average index as in Lobato and Velasco (2000) but using a more recent dataset. Intraday data for the 30 components of this index are obtained from QuantQuote for the period January 3, 2000 to June 06, 2015, i.e. $n = 4393$ daily observations. As trading volume appears to be characterized by nonlinear time trends (see e.g. Mougoué and Aggarwal 2011, Gallant et al. 1993), we use MATLAB's polynomial curve fitting algorithm to remove the nonlinear deterministic trends up to cubic forms prior to proceeding with the two steps of the main analysis.

4.2. Power law in coherency analysis

The first step of the empirical analysis consists in detecting whether the system is characterized by power law in coherency by studying the significance of $\hat{\delta}_\rho$. The results reported in Table 1 include the estimated parameter and its associated bootstrapped 95% confidence interval for all firms and realized measures of variance, where the latter is computed by following the same procedure as Arteche and Orbe (2016), which is detailed in Subsection 2.4.

First of all, the negative sign of $\hat{\delta}_\rho$ indicates that the cross-spectrum long memory estimator $\hat{\delta}_{12}$ is smaller than $\hat{\delta}_1$ and $\hat{\delta}_2$ in almost all cases. Most importantly, up to 60% of the total number of firms verify the conditions of anticointegration (i.e., a negative and statistically significant power law in coherency estimator). At this stage, the results are in line with the findings of Lobato and Velasco (2000), Fleming and Kirby (2011) and Rossi and Santucci de Magistris (2013a) in the sense that they confirm the failure of the cointegration hypothesis and tend to invalidate the findings of Bollerslev and Jubinski (1999).

Table 1: Power law in coherency analysis with $m = \lfloor n^{0.7} \rfloor$

Firm	RV		RK		BPV		medRV	
	$\hat{\delta}_\rho$	BCI(δ_ρ, 95%)	$\hat{\delta}_\rho$	BCI(δ_ρ, 95%)	$\hat{\delta}_\rho$	BCI(δ_ρ, 95%)	$\hat{\delta}_\rho$	BCI(δ_ρ, 95%)
Apple	- 0.118	(-0.243; -0.050)	- 0.133	(-0.316; -0.082)	- 0.122	(-0.303; -0.059)	- 0.120	(-0.227; -0.096)
A.E.	- 0.228	(-0.448; -0.155)	- 0.237	(-0.426; -0.114)	- 0.224	(-0.431; -0.131)	- 0.228	(-0.438; -0.117)
Boeing	- 0.116	(-0.318; -0.043)	- 0.112	(-0.234; -0.033)	- 0.109	(-0.311; -0.010)	- 0.102	(-0.326; -0.022)
Caterpillar	- 0.069	(-0.299; -0.016)	- 0.077	(-0.233; -0.006)	- 0.062	(-0.264; 0.035)	- 0.084	(-0.297; -0.024)
Cisco	- 0.184	(-0.306; -0,115)	- 0.172	(-0.288; -0.066)	- 0.181	(-0.300; -0.096)	- 0.188	(-0.325; -0.164)
Chevron	- 0.183	(-0.396; -0.097)	- 0.142	(-0.330; 0.004)	- 0.179	(-0.383; -0.100)	- 0.180	(-0.394; -0.099)
Dupont	- 0.063	(-0.300; 0.003)	- 0.061	(-0.285; 0.007)	- 0.064	(-0.308; 0.000)	- 0.060	(-0.285; 0.007)
Walt Disney	- 0.137	(-0.307; -0.115)	- 0.170	(-0.387; -0.076)	- 0.139	(-0.315; -0.115)	- 0.132	(-0.303; -0.104)
General Electric	- 0.217	(-0.403; -0,108)	- 0.251	(-0.449; -0.131)	- 0.222	(-0.407; -0.115)	- 0.220	(-0.396; -0.110)
Goldman Sachs	- 0.113	(-0.249; -0.061)	- 0.119	(-0.254; -0.063)	- 0.114	(-0.247; -0.060)	- 0.112	(-0.238; -0.051)
Home Depot	- 0.267	(-0.420; -0,181)	- 0.279	(-0.502; -0.198)	- 0.258	(-0.427; -0.233)	- 0.259	(-0.426; -0.227)
IBM	- 0.146	(-0.350; -0.078)	- 0.115	(-0.238; -0.093)	- 0.147	(-0.350; -0.072)	- 0.150	(-0.367; -0.079)
Intel Corp.	- 0.028	(-0.360; 0.157)	- 0.027	(-0.334; 0.123)	- 0.034	(-0.370; 0.251)	- 0.025	(-0.357; 0.251)
Johnson	- 0.058	(-0.295; 0.009)	- 0.081	(-0.217; -0.044)	- 0.072	(-0.230; 0.012)	- 0.076	(-0.254; 0.024)
JP Morgan	- 0.204	(-0.391; -0,101)	- 0.211	(-0.421; -0.120)	- 0.200	(-0.389; -0.096)	- 0.193	(-0.383; -0.087)
Coca-Cola	- 0.072	(-0.309; 0.004)	- 0.078	(-0.308; 0.014)	- 0.072	(-0.312; 0.001)	- 0.074	(-0.301; -0.003)
McDonald	- 0.270	(-0.409; -0.187)	- 0.402	(-0.615; -0.310)	- 0.265	(-0.398; -0.168)	- 0.269	(-0.405; -0.182)
3M Co.	- 0.085	(-0.304; -0.029)	- 0.074	(-0.295; -0.003)	- 0.086	(-0.296; -0.025)	- 0.089	(-0.300; -0.018)
Merk	- 0.111	(-0.294; -0.034)	- 0.211	(-0.404; -0.077)	- 0.106	(-0.299; -0.029)	- 0.105	(-0.318; -0.033)
Microsoft	- 0.010	(-0.281; 0.088)	0.022	(-0.272; 0.132)	- 0.006	(-0.296; 0.068)	- 0.002	(-0.317; 0.077)
Nike	- 0.109	(-0.352; -0.003)	- 0.120	(-0.352; -0.000)	- 0.090	(-0.345; 0.008)	- 0.102	(-0.344; 0.002)
Pfizer	- 0.060	(-0.184; -0.001)	- 0.051	(-0.262; 0.033)	- 0.042	(-0.236; 0.034)	- 0.042	(-0.349; 0.054)
P&G	- 0.075	(-0.215; -0.049)	- 0.073	(-0.214; -0.055)	- 0.090	(-0.320; -0.025)	- 0.051	(-0.255; 0.038)
The Travelers	- 0.018	(-0.258; 0.075)	- 0.055	(-0.296; 0.038)	- 0.026	(-0.241; 0.048)	- 0.023	(-0.271; 0.044)
UnitedHealth	- 0.059	(-0.197; 0.025)	- 0.047	(-0.199; -0.015)	- 0.051	(-0.269; 0.027)	- 0.051	(-0.281; 0.023)
United Tech.	- 0.101	(-0.258; -0.087)	- 0.110	(-0.321; -0.052)	- 0.105	(-0.255; -0.098)	- 0.101	(-0.260; -0.085)
Verizon	- 0.106	(-0.309; -0.019)	- 0.103	(-0.252; -0.015)	- 0.106	(-0.299; 0.001)	- 0.095	(-0.306; -0.020)
Visa	0.057	(-0.502; 0.382)	- 0.022	(-0.527; 0.349)	0.068	(-0.496; 0.395)	0.074	(-0.486; 0.400)
Wal-Mart	- 0.156	(-0.278; -0.074)	- 0.172	(-0.371; -0.063)	- 0.169	(-0.370; -0.086)	- 0.168	(-0.376; -0.091)
Exxon Mobil	- 0.174	(-0.393; -0.094)	- 0.137	(-0.327; -0.011)	- 0.175	(-0.391; -0.09)	- 0.176	(-0.392; -0.094)

Notes: We use the procedure of Sela and Hurvich (2012) to estimate δ_{12} and to compute $\hat{\delta}_\rho$. Column BCI(δ_ρ,95%) presents the associated 95% bootstrapped confidence interval computed by following the procedure of Arteche and Orbe (2016). The analysis is performed for the four realized measures of volatility, namely RV, RK, BPV, and medRV, respectively.

Nonetheless, our results are clearly original because they reveal that the idiosyncratic persistent components of trading volume and volatility series dwarf a less persistent common factor. Neither cointegration nor traditional short-run techniques can detect this common factor, but one can presume that it is related to the information-arrival process, as in Bollerslev and Jubinski (1999). Interestingly, the results are robust to the choice of the realized measure, thereby revealing that jumps do not affect the cross-spectrum around the frequencies at which the power-law coherency occurs. This is not particularly surprising because one can expect that jumps affect the spectrum at higher frequencies. In some sense, our findings are jump-robust because our semi-parametric frequency domain approach relies on local Whittle estimators that are robust to additive noise. Our findings are also in line with Giot et al. (2010) who demonstrate that trading volume and volatility are positively related essentially through the continuous component of volatility.[5]

4.3. Phase spectrum analysis

We pursue our analysis with the investigation of the phase behaviour near zero frequency by applying the methodology described in Subsection 2.4. Recall that if one detects the presence of significant lead-lag effects the SAIH is supported, whereas the MDH is favoured in the opposite case.

The results for the RV measure are reported in Table 2.[6] The first column indicates the ordinary frequency ($\lambda^*/2\pi$) at which power-law coherency occurs, with λ^* the frequency at which the squared coherency is maximal at low frequencies, i.e. in the vicinity of the origin. To compute the cross-spectrum, we use Welch's method associated with a modified Bartlett-Hann window. This approach reduces not only the estimation bias but also the sample size, so that the near-zero frequencies are shifted to 0. Consequently, a power-law occurring too close to the origin is not identifiable anymore. In such a case, power-law in coherency and cointegration phenomena are indistinguishable for numerical reasons. These inconclusive cases are denoted by a tag '-' instead of a non-null frequency in Table 2 and in Tables 5 to 7 available in the Appendix. It follows that one cannot conclude against the MDH for the 12 firms for which this situation arises.

In the sequel we study the 18 remaining firms. Given the rejection of the cointegration hypothesis documented in Table 1 and supported by the sensitivity analyses that are available in Section 6 and the negative values of $\hat{\delta}_\rho$, one can reasonably conclude in favour of power law coherency for most of these firms. The next 3 columns report the squared coherency, $\rho^2(\lambda^*)$, and its 95% confidence bounds. For all firms and all realized measures, $\rho^2(\lambda^*)$ is significantly different from 0 and generally greater than 0.6, revealing a strong degree of dependence between the two variables. The last 3 columns contain the phase parameter, $\varphi(\lambda^*)$, as well as its 95% confidence bounds. These results indicate whether or not

[5]Unreported sensitivity checks also reveal the robustness of our results to bandwidth selection.

[6]Similar analyses are performed for the other measures of volatility, the results being summarized in Tables 6, 5 and 7 in the Appendix. All results are quantitatively and qualitatively similar.

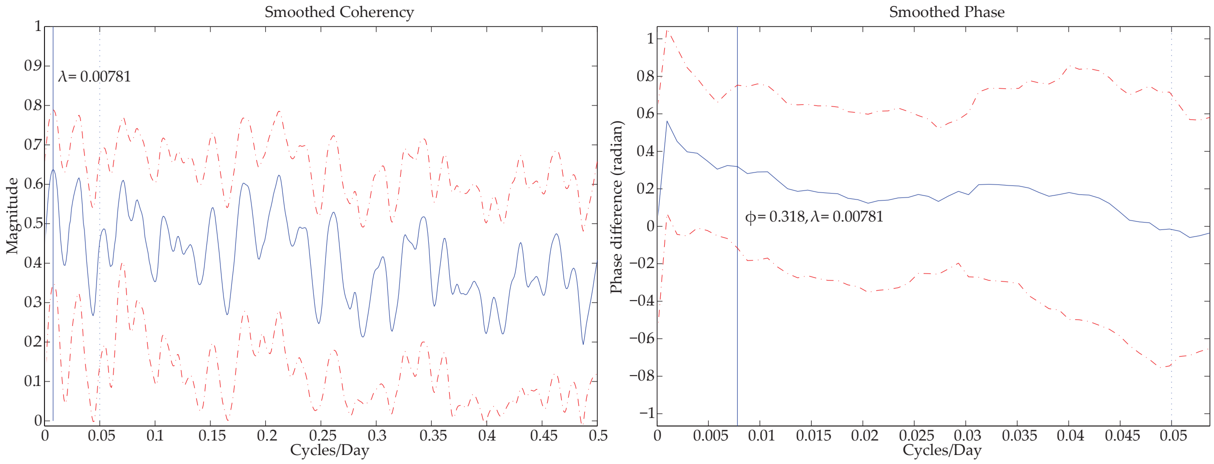
Table 2: Coherency and phase analyses for the RV measure

Firm	$\lambda^*/2\pi$	$\rho_l^2(\lambda^*)$	$\rho^2(\lambda^*)$	$\rho_u^2(\lambda^*)$	$\varphi_l(\lambda^*)$	$\varphi(\lambda^*)$	$\varphi_u(\lambda^*)$
Apple	0.0078	0.3471	0.6376	0.7892	-0.1161	0.3184	0.7529
A.E.	-	0.5262	0.7871	0.8759	-	-	-
Boeing	0.0283	0.2734	0.6069	0.7706	-0.5178	-0.0570	0.4038
Caterpillar	0.0127	0.3732	0.6703	0.8131	-0.3740	-0.0021	0.3699
Cisco	0.0127	0.5064	0.7624	0.8717	-0.4190	-0.0696	0.2797
Chevron	0.0127	0.4342	0.7126	0.8412	-0.2280	0.1103	0.4486
Dupont	0.0127	0.3963	0.7010	0.8363	-0.2513	0.3022	0.8440
Walt Disney	-	0.4722	0.7450	0.8375	-	-	-
General Electric	-	0.6197	0.8404	0.9093	-	-	-
Goldman Sachs	-	0.6918	0.8548	0.9185	-	-	-
Home Depot	-	0.5317	0.8650	0.9145	-	-	-
IBM	0.0127	0.4753	0.7423	0.8582	-0.2341	0.0678	0.3697
Intel Corp.	0.0283	0.3442	0.6890	0.8211	-0.2925	0.1539	0.6003
Johnson	-	0.4344	0.7446	0.8413	-	-	-
JP Morgan	0.0127	0.6329	0.8253	0.9066	-0.2661	0.0583	0.3827
Coca-Cola	-	0.3381	0.6751	0.8047	-	-	-
McDonald	-	0.5523	0.8148	0.8924	-	-	-
3M Co.	-	0.3575	0.6866	0.8233	-	-	-
Merk	0.0283	0.3540	0.6656	0.8101	-0.4171	-0.0048	0.4075
Microsoft	-	0.2876	0.6158	0.7787	-	-	-
Nike	0.0020	0.1513	0.5340	0.7278	-0.2638	0.1558	0.5755
Pfizer	0.0273	0.3694	0.6640	0.8074	-0.4628	-0.0022	0.4583
P&G	-	0.5137	0.7398	0.8314	-	-	-
The Travelers	0.0078	0.1574	0.4471	0.6698	-0.5335	0.0125	0.5586
UnitedHealth	0.0146	0.2775	0.6129	0.7759	-0.2745	0.3547	0.9839
United Tech.	0.0127	0.3954	0.6728	0.8158	-0.2301	0.2225	0.6752
Verizon	0.0137	0.3352	0.6490	0.7967	-0.3190	0.1856	0.6901
Visa	0.0371	0.4021	0.7583	0.8780	-0.4465	0.0766	0.5998
Wal-Mart	-	0.4929	0.7420	0.8541	-	-	-
Exxon Mobil	0.0283	0.5538	0.7897	0.8859	-0.3094	0.0026	0.3146

Notes: $\rho^2(\lambda^*)$ is the squared coherency evaluated at λ^* and $\rho_l^2(\lambda^*)$ and $\rho_u^2(\lambda^*)$ are the bounds of its 95% confidence interval. $\varphi(\lambda^*)$ is the phase difference evaluated at λ^* and $\varphi_l(\lambda^*)$ and $\varphi_u(\lambda^*)$ are the bounds of its 95% confidence interval. When the frequency $\lambda^*/2\pi \to 0$, power-law in coherency and cointegration phenomena are indistinguishable for numerical reasons and a tag '-' is reported.

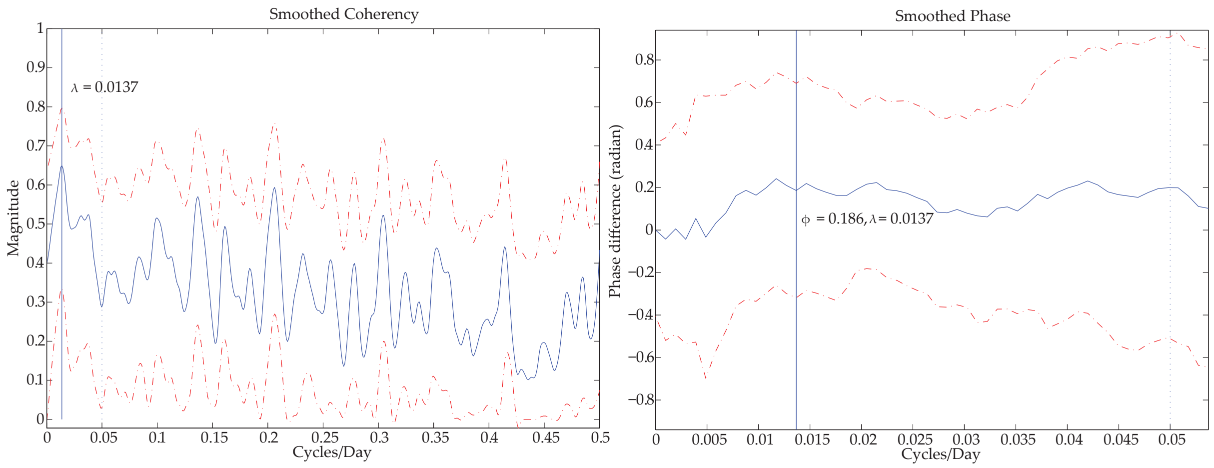
trading volume and volatility series move in phase at frequency $\lambda^* > 0$ where the squared coherency is significantly different from zero and reasonably high. Overall, our findings provide strong evidence in favour of the MDH. This is the case for the 13 firms (using RV), 12 (using RK), and 11 (under BPV and medRV) out of the 18 under analysis at this stage, for which φ is not significantly different from zero. In other words, the two variables are only contemporaneously correlated with a positive sign and we can reject both the hypothesis of a pro-cyclical effect from one variable to another and that of an anti-phase co-movement, i.e. a negative correlation.

Figure 2: Smoothed squared coherency and phase spectra of realized variance and volume for Apple.



Notes: The dashed red lines delimit the 95% confidence interval of coherency and phase, respectively. The vertical solid line represents $\lambda^*/2\pi$. To improve the visibility of the phase at low frequencies, the right panel zooms on the phase difference over the subinterval $\lambda^*/2\pi \in [0, 0.05]$. The vertical dashed blue line depicts the 0.05 bound of this interval in the left panel plot.

Figure 3: Smoothed squared coherency and phase spectra of the realized variance and volume for Verizon.



Notes: The dashed red lines delimit the 95% confidence interval of coherency and phase, respectively. The vertical solid line represents $\lambda^*/2\pi$. To improve the visibility of the phase at low frequencies, the right panel zooms on the phase difference over the subinterval $\lambda^*/2\pi \in [0, 0.05]$. The vertical dashed blue line depicts the 0.05 bound of this interval in the left panel plot.

To give more intuition regarding the power law behaviour in the co-spectrum and the long run phase analysis, two illustrative cases are displayed in Figures 2 and 3, i.e. for Apple and Verizon. Interestingly, the left panels show that the squared coherency, $\rho^2(\lambda)$, is generally significantly different from 0 at all frequencies although a slight downward trend is evidenced. This general pattern might explain why the literature provides mixed results. Indeed, the volume-volatility relationship seems to materialize at many frequencies both in the short and the long run. However, it often excludes the cointegration case, i.e. when $\lambda \to 0$, whereas in the short run the squared coherency behaves more erratically and strongly depends on the firm considered. Our analysis also corroborates the findings of Lobato and Velasco (2000), i.e. $\rho^2(0)$ below 0.4. Indeed, Figures 2 and 3 reveal that at the origin the smoothed coherency does not often go beyond this threshold, zero generally being included in its confidence interval. Finally, regarding the phase difference dynamics, the right panels of the two figures show that zero is always inside the confidence bands, revealing that the MDH is strongly supported at low frequencies ($\lambda^*/2\pi < 0.05$).

5. Robustness analysis

In this section we look at the robustness of our findings from a different perspective than the sensitivity of the power law in coherency and phase spectrum estimates to the various volatility proxies previously discussed. In fact we argue that as our power law coherency analysis strongly rejects the cointegration hypothesis, we should be able to confirm this result by directly testing for the absence of cointegration.

We proceed within a two-step conditional testing framework. First, as the equality of integration orders of two series is a necessary but not sufficient condition for the presence of cointegration, in a first stage of our robustness analysis we test for this hypothesis. We apply the procedure of Hualde (2013) because its test statistic is simple to implement and under the null hypothesis of equality of integration orders of volatility and volume, i.e. $H_0^H : \delta_{rm} = \delta_{vo}$, converges to a standard normal distribution $\mathcal{N}(0, 1)$. As in the main analysis, the estimates of δ_{rm} and δ_{vo} are obtained from the local polynomial Whittle (LPW) estimator of Frederiksen et al. (2012).

Subsequently, in the cases where the null hypothesis of the first test, H_0^H, is not rejected, the cointegration hypothesis can be tested. For this, we use the regression-based fractional cointegration testing procedure proposed by Wang et al. (2015). In the spirit of Hualde (2013), this test requires only a prior estimation of the integration orders of the observed series and of the estimated residuals of the long run regression. Assuming a simple linear model to describe the long-run relationship between rm_t and vo_t,

$$rm_t = \beta vo_t + \varepsilon_t,$$

cointegration theory applies as long as $\beta \neq 0$ and $\varepsilon_t \sim I(\delta_\varepsilon)$ with $\delta_\varepsilon < (\delta_{vo} = \delta_{rm})$. We estimate β by using the fully modified narrow-band least squares approach of Nielsen and Frederiksen (2011), which has been shown to be somewhat robust to endogeneity bias.[7] Then, we collect the residuals $\hat{\varepsilon}_t$ and estimate δ_ε by applying the LPW estimator. Finally, we compute the test statistic suggested by Wang et al. (2015) by using $\hat{\delta}_{vo} = \hat{\delta}_{rm}$ and $\hat{\delta}_\varepsilon$. Inference is simple as the test statistic was shown to be asymptotically standard

[7] To apply this estimator several bandwidths have to be selected. Following the terminology of Nielsen and Frederiksen (2011), we set $m_1 = \lfloor n^{0.7} \rfloor$, $m_2 = \lfloor n^{0.7} \rfloor$, $m_3 = \lfloor n^{0.3} \rfloor$, $m_0 = m_3$.

normal under the null hypothesis of no fractional cointegration, $H_0^W : \hat{\delta}_\varepsilon = (\delta_{vo} = \delta_{rm})$, and to diverge under the alternative.

The results of the two tests, of integration order equality and cointegration, respectively, are reported in Table 4. '0' corresponds to the non-rejection of the null hypothesis of each test, while '1' indicates the opposite. A ('0', '1') couple is therefore an indicator of presence of cointegration. A tag '/' is reported for the second test when the equality of integration orders is not verified, as testing for cointegration does not make sense in such a case. The results are in line with the findings of Lobato and Velasco (2000), Fleming and Kirby (2011) and Rossi and Santucci de Magistris (2013a) in the sense that they confirm the failure of the cointegration hypothesis in nearly all cases and tend to invalidate the findings of Bollerslev and Jubinski (1999). Indeed, for only four firms the null of absence of fractional cointegration is systematically rejected regardless of the realized measure considered.

6. Conclusion

In this paper, we investigate the nature of the relationship between trading volume and volatility for the thirty components of the Dow Jones stock market index. In line with the existing literature, we scrutinize whether the MDH or the SAIH receive more empirical support. But contrary to previous studies, which generally focus either on the long-run or on the short-run, we chose to work with a very recent and original p-component model that encompasses various possible long-run behaviours: fractional cointegration, anti-cointegration or absence of common dynamics. In particular, our approach can detect a power law coherency away from the zero frequency, which reveals that the two series are connected at medium frequencies in more than half of the firms under analysis. Said otherwise, the idiosyncratic, persistent nature of trading volume and volatility dwarfs a less persistent, hidden, common factor that can be linked with the theoretical financial literature on the information arrival process. A subsequent phase spectrum analysis reveals that volatility and volume are characterized by contemporaneous positive dependence at low frequencies. In absence of significant lead-lag effects, we hence conclude in favour of the mixture of distributions hypothesis for all the firms under analysis.

Table 3: Long memory estimates for RV, RK, BPV, and $medRV$ with $m = \lfloor n^{0.7} \rfloor$

	δ_{rm}	δ_{vo}	δ_{rm}	δ_{vo}	δ_{rm}	δ_{vo}	δ_{rm}	δ_{vo}	δ_{rm}	δ_{vo}
	Apple		A.E.		Boeing		Caterpillar		Cisco	
RV	0.4802	0.5596	0.6628	0.7034	0.5718	0.4505	0.5720	0.4018	0.5716	0.5571
	(0.3338;0.5762)	(0.3875;0.699)	(0.4488;0.8778)	(0.5144;0.891)	(0.3899;0.7766)	(0.2954;0.6602)	(0.3774;0.7962)	(0.2505;0.6457)	(0.4416;0.6603)	(0.415;0.6448)
RK	0.4948		0.6677		0.5454		0.5694		0.5411	
	(0.3522;0.6761)		(0.4139;0.8373)		(0.3876;0.6404)		(0.3948;0.6596)		(0.3466;0.6382)	
BPV	0.4896		0.6552		0.5603		0.5581		0.5572	
	(0.3343;0.6763)		(0.4146;0.8434)		(0.3589;0.7537)		(0.3294;0.7456)		(0.4005;0.6457)	
medRV	0.4885		0.6629		0.5499		0.6066		0.5732	
	(0.4616;0.5575)		(0.4163;0.8619)		(0.3623;0.7643)		(0.4253;0.8063)		(0.4836;0.6565)	
	Chevron		Dupont		Walt Disney		General Electric		Goldman Sachs	
RV	0.6272	0.5994	0.5923	0.3556	0.5997	0.4785	0.6298	0.6948	0.5768	0.5101
	(0.4113;0.8095)	(0.4099;0.7966)	(0.4049;0.7896)	(0.2359;0.6331)	(0.5895;0.6876)	(0.3724;0.5997)	(0.4165;0.8092)	(0.458;0.8565)	(0.4244;0.6744)	(0.3494;0.6011)
RK	0.5284		0.5741		0.6440		0.6888		0.5818	
	(0.2756;0.7138)		(0.3951;0.7812)		(0.4284;0.8427)		(0.4359;0.8843)		(0.4321;0.694)	
BPV	0.6169		0.5874		0.6107		0.6413		0.5787	
	(0.4342;0.8095)		(0.3959;0.7704)		(0.5875;0.6978)		(0.4108;0.8155)		(0.4361;0.6711)	
medRV	0.6202		0.5812		0.6032		0.6384		0.5751	
	(0.4336;0.8109)		(0.3931;0.7775)		(0.5231;0.6868)		(0.409;0.7976)		(0.4172;0.6675)	
	Home Depot		IBM		Intel Corp.		Johnson		JP Morgan	
RV	0.6201	0.7507	0.6066	0.5388	0.5823	0.1980	0.5505	0.3922	0.6649	0.6476
	(0.4463;0.7306)	(0.5495;0.8402)	(0.4206;0.774)	(0.3617;0.7302)	(0.403;0.7771)	(-0.1626;0.7676)	(0.3606;0.7694)	(0.2275;0.6195)	(0.4546;0.8226)	(0.4022;0.8091)
RK	0.6226		0.5521		0.5513		0.5700		0.6681	
	(0.4949;0.8099)		(0.4356;0.63)		(0.3928;0.7657)		(0.4777;0.6504)		(0.4344;0.8633)	
BPV	0.6010		0.6094		0.5845		0.5782		0.6556	
	(0.5886;0.6908)		(0.43;0.78)		(0.3474;0.7555)		(0.3995;0.6596)		(0.4321;0.8156)	
medRV	0.6040		0.6157		0.5739		0.5882		0.6443	
	(0.5649;0.6965)		(0.43;0.7873)		(0.3311;0.7472)		(0.4061;0.673)		(0.4378;0.8032)	
	Coca-Cola		McDonald		3M Co.		Merk		Microsoft	
RV	0.6115	0.3495	0.5766	0.7956	0.5915	0.4410	0.5810	0.3703	0.5992	0.2372
	(0.4269;0.805)	(0.1975;0.6277)	(0.4085;0.6704)	(0.6068;0.8942)	(0.4022;0.7919)	(0.2922;0.6344)	(0.4187;0.7931)	(0.2171;0.5817)	(0.4178;0.7916)	(-0.0092;0.6509)
RK	0.5959		0.8222		0.5597		0.7455		0.5116	
	(0.3794;0.8116)		(0.6101;1)		(0.3765;0.7606)		(0.4837;0.9757)		(0.329;0.733)	
BPV	0.6130		0.5680		0.5950		0.5781		0.5891	
	(0.4398;0.819)		(0.384;0.6527)		(0.407;0.7994)		(0.4039;0.7829)		(0.4191;0.7656)	
medRV	0.6147		0.5765		0.6066		0.5797		0.5834	
	(0.4529;0.8209)		(0.4088;0.6601)		(0.4209;0.805)		(0.4012;0.77)		(0.4095;0.7605)	
	Nike		Pfizer		P&G		The Travelers		UnitedHealth	
RV	0.6693	0.2938	0.5666	0.3569	0.5541	0.4238	0.5832	0.3208	0.5748	0.3597
	(0.4613;0.8763)	(0.0935;0.6161)	(0.4212;0.6483)	(0.2164;0.4397)	(0.4506;0.6346)	(0.313;0.5083)	(0.4002;0.6901)	(0.0839;0.6319)	(0.41;0.6615)	(0.1501;0.4825)
RK	0.6530		0.5215		0.5261		0.6319		0.5285	
	(0.4353;0.8506)		(0.335;0.724)		(0.4502;0.6065)		(0.4118;0.8482)		(0.429;0.6138)	
BPV	0.6360		0.5354		0.5890		0.5942		0.5586	
	(0.433;0.8405)		(0.3643;0.7119)		(0.421;0.7873)		(0.4231;0.6982)		(0.3823;0.7684)	
medRV	0.6578		0.5392		0.5083		0.5938		0.5638	
	(0.4627;0.8666)		(0.3726;0.624)		(0.3065;0.6961)		(0.4145;0.813)		(0.3803;0.7849)	
	United Tech.		Verizon		Visa		Wal-Mart		Exxon Mobil	
RV	0.5658	0.4977	0.5926	0.4172	0.3946	0.0911	0.5752	0.5381	0.6153	0.5877
	(0.4689;0.6434)	(0.3891;0.5769)	(0.3912;0.7922)	(0.2494;0.6537)	(0.2181;0.8841)	(-0.5;1)	(0.4009;0.6685)	(0.3682;0.6328)	(0.4286;0.8063)	(0.4061;0.7909)
RK	0.5779		0.5560		0.5128		0.5887		0.5310	
	(0.3758;0.7782)		(0.3695;0.6928)		(0.2011;0.9068)		(0.3532;0.7953)		(0.2923;0.7146)	
BPV	0.5730		0.5983		0.3853		0.5987		0.6189	
	(0.4908;0.6597)		(0.3814;0.7909)		(0.178;0.869)		(0.414;0.8099)		(0.432;0.7999)	
medRV	0.5685		0.5863		0.3794		0.5987		0.6226	
	(0.4669;0.6557)		(0.3972;0.7923)		(0.1956;0.8386)		(0.4217;0.8)		(0.4319;0.8002)	

Notes: For each firm we report the long memory estimates for volatility and volume, i.e. δ_{rm} and δ_{vo}, obtained by using the procedure of Frederiksen et al. (2012) on four realized measures of volatility, namely RV, RK, BPV, and medRV. 95% percentile bootstrap confidence intervals are reported between brackets.

Table 4: Cointegration analysis and integration orders homogeneity tests for RV, RK, BPV, and $medRV$

Firm	RV		RK		BPV		medRV	
	Test 1	Test 2	Test 1	Test 2	Test 1	Test 2	Test 1	Test 2
Apple	0	0	0	0	0	0	0	0
A.E.	0	0	0	1	0	1	0	1
Boeing	0	0	0	0	0	0	0	0
Caterpillar	0	0	0	0	0	0	1	/
Cisco	0	1	0	1	0	1	0	1
Chevron	0	0	0	0	0	0	0	0
Dupont	1	/	1	/	1	/	1	/
Walt Disney	0	0	1	/	0	0	0	0
General Electric	0	1	0	1	0	1	0	1
Goldman Sachs	0	0	0	0	0	0	0	0
Home Depot	0	1	0	1	0	1	0	1
IBM	0	0	0	0	0	0	0	0
Intel Corp.	1	/	1	/	1	/	1	/
Johnson	0	0	1	/	1	/	1	/
JP Morgan	0	1	0	0	0	1	0	1
Coca-Cola	1	/	1	/	1	/	1	/
McDonald	1	/	0	1	1	/	1	/
3M Co.	0	0	0	0	0	0	0	0
Merk	0	0	1	/	1	/	1	/
Microsoft	1	/	0	0	1	/	1	/
Nike	1	/	1	/	1	/	1	/
Pfizer	1	/	1	/	1	/	1	/
P&G	0	0	0	0	0	0	0	0
The Travelers	1	/	1	/	1	/	1	/
UnitedHealth	1	/	0	0	1	/	1	/
United Tech.	0	0	0	0	0	0	0	0
Verizon	1	/	0	0	1	/	1	/
Visa	0	0	0	0	0	0	0	0
Wal-Mart	0	0	0	0	0	0	0	0
Exxon Mobil	0	0	0	0	0	0	0	0

Notes: We use the procedures of Hualde (2013) and Wang et al. (2015) to test the equality of integration orders (*Test 1*) and the cointegration hypothesis (*Test 2*), respectively. For the first one, we report a value of '1' if there is no equality of integration orders (the null is rejected) and a '0' otherwise. For the latter, a value of '1' is in favor of cointegration (the null is rejected), a value of '0' dismisses it, while a tag "/" means that cointegration does not make sense to be tested since the equality of orders condition was not validated. The analysis is performed for the four measures of volatility considered, namely RV, RK, BPV, and medRV, respectively.

Appendix

Table 5: Coherency and phase analyses for the RK measure

Firm	$\lambda^*/2\pi$	$\rho_l^2(\lambda^*)$	$\rho^2(\lambda^*)$	$\rho_u^2(\lambda^*)$	$\varphi_l(\lambda^*)$	$\varphi_{12}(\lambda^*)$	$\varphi_u(\lambda^*)$
Apple	0.0264	0.3211	0.6395	0.7949	-0.2676	0.1129	0.4934
A.E.	-	0.5301	0.7829	0.8735	-	-	-
Boeing	0.0283	0.2878	0.6277	0.7853	-0.4926	-0.0543	0.3841
Caterpillar	0.0127	0.3385	0.6486	0.8001	-0.4365	-0.0444	0.3477
Cisco	0.0127	0.4628	0.7348	0.8552	-0.4563	-0.0898	0.2767
Chevron	0.0127	0.4077	0.6891	0.8255	-0.2200	0.1209	0.4618
Dupont	0.0127	0.4256	0.7183	0.8453	-0.3051	0.2320	0.7690
Walt Disney	-	0.4734	0.7069	0.8187	-	-	-
General Electric	-	0.6217	0.8221	0.9006	-	-	-
Goldman Sachs	-	0.6962	0.8589	0.9226	-	-	-
Home Depot	-	0.5368	0.8546	0.9078	-	-	-
IBM	0.0264	0.3999	0.7001	0.8328	-0.3850	-0.0065	0.3720
Intel Corp.	0.0283	0.2457	0.5783	0.7443	-0.4081	0.1222	0.6525
Johnson	0.0137	0.3930	0.7029	0.8321	-0.3234	0.1360	0.5954
JP Morgan	0.0137	0.6094	0.8135	0.9000	-0.2460	0.0885	0.4229
Coca-Cola	-	0.3221	0.6699	0.8032	-	-	-
McDonald	-	0.4977	0.7946	0.8855	-	-	-
3M Co.	-	0.4098	0.7070	0.8370	-	-	-
Merk	0.0283	0.3437	0.6498	0.7985	-0.3595	0.0614	0.4823
Microsoft	0.0137	0.2760	0.6090	0.7747	-0.2423	0.2317	0.7057
Nike	-	0.1626	0.5462	0.7372	-	-	-
Pfizer	0.0264	0.4326	0.7175	0.8405	-0.4203	0.0296	0.4794
P&G	-	0.4795	0.7061	0.8223	-	-	-
The Travelers	0.0186	0.1345	0.4465	0.6656	-0.8107	-0.0643	0.6821
UnitedHealth	0.0137	0.3202	0.6028	0.7664	-0.4291	0.2371	0.9033
United Tech.	0.0068	0.3485	0.6613	0.8032	-0.0873	0.2537	0.5947
Verizon	0.0283	0.3660	0.6669	0.8114	-0.3905	-0.0274	0.3358
Visa	0.0371	0.5412	0.8351	0.9244	-0.5455	0.0011	0.5477
Wal-Mart	-	0.4449	0.7000	0.8272	-	-	-
Exxon Mobil	0.0283	0.6250	0.8272	0.9073	-0.2177	0.0469	0.3115

Notes: $\rho^2(\lambda^*)$ is the squared coherency evaluated at λ^* and $\rho_l^2(\lambda^*)$ and $\rho_u^2(\lambda^*)$ are the bounds of its 95% confidence interval. $\varphi(\lambda^*)$ is the phase difference evaluated at λ^* and $\varphi_l(\lambda^*)$ and $\varphi_u(\lambda^*)$ are the bounds of its 95% confidence interval. When the frequency $\lambda^*/2\pi \to 0$, power-law in coherency and cointegration phenomena are indistinguishable for numerical reasons and a tag '-' is reported.

Table 6: Coherency and phase analyses for the BPV measure

Firm	$\lambda^*/2\pi$	$\rho_l^2(\lambda^*)$	$\rho^2(\lambda^*)$	$\rho_u^2(\lambda^*)$	$\varphi_l(\lambda^*)$	$\varphi_{12}(\lambda^*)$	$\varphi_u(\lambda^*)$
Apple	0.0283	0.3270	0.6441	0.7958	-0.2446	0.1397	0.5239
A.E.	-	0.5277	0.7886	0.8766	-	-	-
Boeing	0.0283	0.2943	0.6130	0.7754	-0.4853	-0.0564	0.3725
Caterpillar	0.0137	0.3820	0.6760	0.8161	-0.3731	0.0161	0.4053
Cisco	0.0127	0.5106	0.7649	0.8731	-0.4455	-0.0908	0.2639
Chevron	0.0127	0.4808	0.7404	0.8576	-0.2472	0.0856	0.4183
Dupont	0.0127	0.3963	0.6952	0.8331	-0.2591	0.2952	0.8245
Walt Disney	-	0.4624	0.7753	0.8493	-	-	-
General Electric	-	0.6281	0.8474	0.9131	-	-	-
Goldman Sachs	-	0.6891	0.8534	0.9173	-	-	-
Home Depot	-	0.5345	0.9068	0.9398	-	-	-
IBM	0.0127	0.4880	0.7485	0.8619	-0.2423	0.0656	0.3734
Intel Corp.	0.0283	0.3333	0.6648	0.8064	-0.2982	0.1766	0.6496
Johnson	-	0.4396	0.7461	0.8499	-	-	-
JP Morgan	0.0127	0.6418	0.8313	0.9098	-0.2754	0.0534	0.3823
Coca-Cola	-	0.3342	0.6729	0.8042	-	-	-
McDonald	-	0.5396	0.8020	0.8813	-	-	-
3M Co.	-	0.3744	0.6962	0.8297	-	-	-
Merk	0.0283	0.3395	0.6575	0.8065	-0.3988	0.0550	0.5088
Microsoft	-	0.2984	0.6283	0.7867	-	-	-
Nike	0.0029	0.1773	0.5445	0.7355	-0.2184	0.1520	0.5225
Pfizer	0.0283	0.3725	0.6708	0.8088	-0.6098	-0.0950	0.4197
P&G	-	0.5144	0.7450	0.8362	-	-	-
The Travelers	0.0078	0.1536	0.4604	0.6792	-0.5442	-0.0070	0.5302
UnitedHealth	0.0137	0.2704	0.6065	0.7721	-0.3148	0.4266	1.1680
United Tech.	0.0127	0.3675	0.6458	0.8000	-0.2425	0.2182	0.6789
Verizon	0.0137	0.3127	0.6330	0.7884	-0.3380	0.1742	0.6864
Visa	0.0371	0.3664	0.7435	0.8756	-0.4700	0.0720	0.6140
Wal-Mart	-	0.4973	0.7447	0.8555	-	-	-
Exxon Mobil	0.0283	0.5186	0.7678	0.8729	-0.3335	-0.0170	0.2994

Notes: see Table 5.

Table 7: Coherency and phase analyses for the medRV measure

Firm	$\lambda^*/2\pi$	$\rho_l^2(\lambda^*)$	$\rho^2(\lambda^*)$	$\rho_u^2(\lambda^*)$	$\varphi_l(\lambda^*)$	$\varphi_{12}(\lambda^*)$	$\varphi_u(\lambda^*)$
Apple	0.0283	0.3171	0.6344	0.7894	-0.2558	0.1333	0.5225
A.E.	-	0.5253	0.7796	0.8703	-	-	-
Boeing	0.0283	0.2875	0.6204	0.7810	-0.5066	-0.0600	0.3866
Caterpillar	0.0137	0.3602	0.6651	0.8097	-0.3515	0.0353	0.4221
Cisco	0.0127	0.5080	0.7626	0.8724	-0.4660	-0.1007	0.2646
Chevron	0.0127	0.4623	0.7290	0.8507	-0.2418	0.0945	0.4309
Dupont	0.0127	0.3912	0.6913	0.8308	-0.2550	0.2823	0.7975
Walt Disney	-	0.4686	0.7294	0.8242	-	-	-
General Electric	-	0.6258	0.8393	0.9095	-	-	-
Goldman Sachs	-	0.6900	0.8583	0.9194	-	-	-
Home Depot	-	0.5396	0.9014	0.9362	-	-	-
IBM	0.0127	0.4745	0.7395	0.8564	-0.2393	0.0732	0.3858
Intel Corp.	0.0283	0.3183	0.6658	0.8083	-0.2782	0.1798	0.6378
Johnson	-	0.4301	0.7333	0.8380	-	-	-
JP Morgan	0.0127	0.6535	0.8379	0.9138	-0.2780	0.0476	0.3732
Coca-Cola	-	0.3490	0.6833	0.8092	-	-	-
McDonald	-	0.5386	0.7971	0.8783	-	-	-
3M Co.	-	0.3738	0.6969	0.8296	-	-	-
Merk	0.0283	0.3128	0.6423	0.7975	-0.3939	0.0668	0.5275
Microsoft	-	0.3039	0.6313	0.7884	-	-	-
Nike	0.0020	0.1534	0.5343	0.7280	-0.2725	0.1506	0.5738
Pfizer	0.0283	0.3604	0.6530	0.7981	-0.5868	-0.0774	0.4320
P&G	-	0.5187	0.7399	0.8344	-	-	-
The Travelers	0.0088	0.1433	0.4630	0.6811	-0.4883	0.0165	0.5214
UnitedHealth	0.0137	0.2971	0.6311	0.7870	-0.3108	0.4089	1.1285
United Tech.	0.0127	0.3650	0.6433	0.7986	-0.2749	0.1889	0.6528
Verizon	0.0137	0.3184	0.6393	0.7915	-0.3471	0.1592	0.6656
Visa	0.0391	0.3146	0.7080	0.8569	-0.5576	0.0234	0.6043
Wal-Mart	-	0.5112	0.7523	0.8598	-	-	-
Exxon Mobil	0.0283	0.5142	0.7658	0.8718	-0.3457	-0.0237	0.2983

Notes: see Table 5.

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