King U.S. Dollar, Global Risks, and Currency Option Risk Premiums*

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January 21, 2022

Abstract

We investigate how the primacy of the U.S. dollar affects the pricing of risks in the currency options market. Our findings are based on a daily option panel of 15 currencies. This analysis reveals that (i) put risk premiums are reliably negative, implying across-the-board interest in hedging dollar appreciations; (ii) single-name call risk premiums are both positive and (puzzlingly) *negative*; (iii) volatility risk premiums are small or insignificant; and (iv) option risk premiums on investment currencies exceed those of funding currencies during high volatility states. We formalize a theory to understand the empirical properties of currency option risk premiums.

*Seminar participants at Temple University, Adam Smith Business School (University of Glasgow), and the Federal Reserve Board Research seminar provided useful feedback that we incorporated in the paper. We thank Mario Cerrato, John Crosby, Xiaohui Gao, Dilip Madan, George Panayotov, Maria Tito, and Zhaowei Zhang for many helpful conversations. We welcome comments, including references to related papers that we have inadvertently overlooked. All computer codes are available from the authors. The assistance of Zhaowei Zhang is gratefully acknowledged. The views in this paper are the responsibility of the authors and do not necessarily represent those of the Federal Reserve Board or the Federal Reserve System.

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1. Introduction

The role of the U.S. dollar is iconic in the international financial system. The invoicing of trade is concentrated in the U.S. dollar. Central banks lean toward holding U.S. dollar reserves, while U.S. Treasury serve as safe assets and facilitate collateral arrangements in financial transactions. Flow data show that sovereign entities and wealth funds are avid buyers of U.S. assets. At the same time, non-U.S. firms borrow in U.S. dollars and, thus, prefer dollar exposures. The balance sheets of foreign banks manifest dollar liabilities that are essentially at par with U.S. banks. Concerted and targeted withdrawals by U.S. investors often destabilize foreign markets and bring currency devaluations and sudden stops. The price of crude oil and gold are denominated in U.S. dollars, and institutionally weaker economies have black markets for dollars. Taken together, evidence indicates that the U.S. dollar is the king of all currencies.[1]

What are the quantitative implications of the U.S. dollar being the king of all currencies? We frame this question in the context of option risk premiums. The implications for option risk premiums derive from a theoretical framework wherein (i) the United States is differentially affected in bad economic states and (ii) bad states are associated with the appreciation of the U.S. dollar. In our theory, the risk premiums of option claims to the downside and upside are influenced by the time-varying sign of the currency risk premium. Our framework offers flexibility in generating realistic heterogeneity in (option and currency) risk premiums together with mimicking exchange rate volatilities across currencies.

We test the theoretical implications using options data for 15 single-name currencies, including the G10 currencies, the world's largest and most traded currencies, and 6 other currencies. The richness of this options data set stems from its daily availability while maintaining a constant expiration of 30 days. Crucial to drawing reliable inferences, this data set contains many more option expiration cycles than most papers (nearly *4,900* per name over the sample from 2000 to 2019). Moreover, option prices are quoted such that the U.S. dollar is the base currency and the foreign currency is the reference.

[1] We refer the reader to the evidence in Goldberg and Tille (2008), Gourinchas and Rey (2007), Mendoza, Quadrini, and Rull (2009), Hassan (2013), Ivashina, Scharfstein, and Stein (2015), Maggiori (2017), Farhi and Maggiori (2018), Du and Schreger (2014), Maggiori, Neiman, and Schreger (2020), Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Moller (2020), **?**, and Gopinath and Stein (2021).

We employ our currency options panel to answer three questions: (1) What is the nature of risk premiums on downside movements on currencies (that is, their depreciation)? (2) Is there evidence that investors worry about risks that cause the U.S. dollar to depreciate? (3) How significant are the risk premiums for bearing currency volatility risks? Our empirical and theoretical analyses reflect macroeconomic disparities across economies and incorporate state-dependent concerns about insuring currency movements.

Pertinent to our questions are the properties of option risk premiums on investment versus funding currencies during heightened economic uncertainties. These data points shape the demand for the U.S. dollar as a global safe asset and may accordingly affect option premiums. Motivating the theory that underlies currency option premiums, we explore whether variables characterizing macroeconomic disparities among the economies can describe the cross-sectional and time-series variation in currency option excess returns.

Economy pairs associated with positive currency risk premiums display negative put risk premiums that decline at lower strikes. The average risk premium on 10 delta puts is -24.8% (respectively, -23.4%) per month across the 15 currencies (respectively, the G10). Instrumental to our theory, 14 of 15 single-name 10 delta put risk premiums are negative. Our evidence indicates broad reservations about foreign currency depreciations, a feature in line with the U.S. dollar being a central player in the global financial system.

Our data suggest a finding that 10 delta call risk premiums are, on *average*, reliably less negative in comparison to puts and switch sign to be positive for 25 delta calls. We attribute this evidence on call risk premiums to the feature that markets are less concerned about U.S. dollar depreciations than appreciations.[2]

The size of the volatility risk premiums, as reflected in single-name straddle excess returns, is empirically small negative, small positive, or statistically indistinguishable from zero. The global volatility risk premium — constructed as an equal-weighted basket of its 15 constituents — is revealed to be smaller, at -0.7% unconditionally (per month, insignificant). This data dimension has economic content, as the desire to hold U.S. dollars takes many forms in the international financial system, and overall aversion about

[2] Decisive to macrofinance models may be the evidence for the Japanese yen, which manifests a *positive* (negative) risk premium for out-of-the-money puts (calls). Our theory reconciles these empirical outcomes in the context of a significantly negative currency risk premium (that is, investors pay a premium to hold the yen), combined with negative risk premiums for upside movements (that is, U.S. dollar depreciations).

currency volatility, when the U.S. dollar is the base currency, appears insignificant.

Because investment and funding currencies may embed different vulnerabilities to global shocks, we examine option premiums for currency sets obtained by dynamically sorting currencies on interest-rate differentials. This empirical treatment supports a finding that put risk premiums are reliably more negative for investment currencies than for funding currencies. Reversing the pattern, we uncover that call premiums are reliably more negative for funding currencies. The volatility risk premiums are indistinguishable between investment and funding currencies.

If, as theory and the extant literature suggest, the U.S. dollar tends to appreciate in bad states, what can be revealed about currency option premiums aligned with heightened economic uncertainties? Inquiring into this matter, we initiate option positions contingent on belonging to one of five bins ranging from low to high volatility levels. Volatility is measured using the VIX or the global currency VIX (constructed by combining single-name currency VIX). We close this position in 30 days. In our analysis, we gauge heightened uncertainty by a VIX (respectively, the global currency VIX) breakpoint higher than 40% (respectively, 16%). The takeaway is that funding currencies, as opposed to investment currencies, manifest more negative put, call, and volatility risk premiums when the option positions are initiated in high volatility states. We infer that funding currencies experience declines of lesser severity, while still eliciting hedging interests.

There are reasons to think that currency option premiums will not be homogeneous across economy pairs. To isolate common underlying economic mechanisms, we investigate whether (i) interest-rate differentials on five-year government bonds (against the United States), (ii) the quadratic variation in currency returns, (iii) currency returns over a trailing window, and (iv) risk reversals can forecast option premiums. In the panel regression framework, we allow for robust standard errors along with year fixed effects and currency fixed effects. Our treatment indicates that some of these macroeconomic disparity variables (in particular, the quadratic variation) are statistically relevant for forecasting currency option premiums.

Focusing on interpretations, we formalize a theory of option premiums that, to our knowledge, has not been introduced in the context of comparing option premiums across currencies. This theory is characterized in terms of the single-name currency risk premiums and the risk premiums associated with higher-order moments of currency returns. Our theory is amenable to addressing questions like those that follow: Why is the put (call) risk premium for the Japanese yen positive (negative)? Why are the put and call risk premiums both negative for some single names? Which sources underlie the variation in currency option premiums? Our assessment exercises imply that a model with global risk drivers, non-normalities in the currency return distributions, and differential exposures to uncertainties shows promise in mimicking the multidimensional attributes of the options data across single-name currencies.

How do our theory and findings on option premiums connect to the literature? Imperative to Farhi and Gabaix (2016) is the possibility of rare but extreme disasters and the link between disasters and risk premiums. Another theory emphasizes the notion of "the exorbitant privilege" of the United States combined with the safe-haven attributes of the U.S. dollar (e.g., Gourinchas and Rey (2007) and Maggiori (2017)). The work of Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Moller (2020), and Gopinath and Stein (2021) proposes a theory of dominant currency based on global trade and banking system configurations.

Our distinction from extant theories is that investors cover positions in both tails of the currency return distribution, and they are averse to both the depreciation of the foreign currency (put risk premiums can be negative) and the depreciation of the U.S. dollar (call risk premiums can be negative). Viewed from this standpoint, the negative call risk premiums can be puzzling if single-name currencies embed positive currency risk premiums.

To our knowledge, our exercises documenting the implications of the U.S. dollar as the king of all currencies for option risk premiums do not have direct parallels. At the center of our inquiry is a theory of currency option risk premiums, and we explore consistencies with the featured empirical findings. Specifically, our treatment considers phenomena related to option risk premiums, and we uncover the return properties of puts, calls, and straddles across single-name currencies and baskets.

Our theoretical and empirical angles are motivated by the lack of a coherent view on how investors respond to downside and upside uncertainties in currency markets. Inferring risk premiums from option returns helps emphasize salient facts unavailable from examining currency instruments that do not offer Arrow-Debreu like payoffs (that is, forwards and currency swaps). Extant models could be enriched if they are informed by the empirical properties of option risk premiums on investment and funding currencies.

2. Hypotheses about currency option premiums

In this section, we present testable hypotheses for currency option risk premiums that are disciplined by a model. We focus on a framework in which the U.S. dollar appreciates in bad economic states. Instrumental to our investigation is the primacy of the U.S. dollar and the links to currency option risk premiums and macroeconomic disparities across economies. We also describe the currency options data used to test the hypotheses about option premiums. Our departure from existing papers is that the single-name currency option quotes are available each day, but they all expire in 30 days. As a result, our empirical analysis exploits a large number of option expiration cycles.

2.1. Motivating and framing the empirical hypotheses

To frame our hypotheses, we consider a setting that — although simple — is relevant to our findings on currency option premiums. In section 4, we develop an international economy model with stochastic volatility in exchange rate growth and random jumps. Our treatment allows for time-varying probability of large unexpected changes in the pricing kernels. Additionally, we develop expressions for currency option premiums. These expressions have not yet been assimilated in international finance research.

In what follows, $\mathbb{P}$ is the real-world probability measure and $\mathbb{Q}$ is the risk-neutral measure that consistently prices U.S. dollar denominated assets. Additionally, $\mathbb{E}_t^{\mathbb{P}}(\bullet) \equiv \mathbb{E}^{\mathbb{P}}(\bullet|\mathcal{F}_t)$ (respectively, $\mathbb{E}_t^{\mathbb{Q}}(\bullet) \equiv \mathbb{E}^{\mathbb{Q}}(\bullet|\mathcal{F}_t)$) is the expectation under $\mathbb{P}$ (respectively, $\mathbb{Q}$). The filtration $\mathcal{F}_t$ satisfies the usual conditions.

Consider a two-date international economy in which the state space ω contains four states, $\omega =$

$(\omega_1, \omega_2, \omega_3, \omega_4)$. The physical probabilities $p[\omega]$ and the pricing kernels in state ω are as follows:

$$p[\omega] = \begin{pmatrix} \frac{1}{2} - \frac{(1-\hbar)(1+\mho)}{4} \\ \frac{(1-\hbar)\mho}{2} \\ \frac{(1-\hbar)}{2} \\ \frac{1}{2} - \frac{(1-\hbar)(1+\mho)}{4} \end{pmatrix}, \qquad M^{\text{us}}_{t+\tau}[\omega] = \begin{pmatrix} 1+\Psi_{\text{us}} \\ 1 \\ 1 \\ 1-\Psi_{\text{us}} \end{pmatrix}, \quad \text{and} \quad M^{\text{j}}_{t+\tau}[\omega] = \begin{pmatrix} 1+\Psi_{\text{j}} \\ 1 \\ 1 \\ 1-\Psi_{\text{j}} \end{pmatrix}, \qquad (1)$$

where $0 \le \hbar \le 1$, $0 \le \mho \le 1$, $0 \le \Psi_{\text{us}} < 1$, $0 \le \Psi_{\text{j}} < 1$, and $\sum_\omega p[\omega] = 1$. The parameter $\hbar$ reflects the exposure to the extremes, and ω_1 (ω_4) is the most unpleasant (pleasant) state. Set $M^{\text{us}}_t = 1$ and $M^{\text{j}}_t = 1$.

Our parameterizations keep the means of $M^{\text{us}}_{t+\tau}$ and $M^{\text{j}}_{t+\tau}$ equal to unity. Thus, the interest rates are zero and the forward exchange rate satisfies $F^{\text{us|j}}_{t,\tau} = S^{\text{us|j}}_t \frac{1+0}{1+0} = 1$ (here, we set $\tau = 1$).

We explore the implications of assuming that $\Psi_{\text{us}} > \Psi_{\text{j}}$. This restriction implies that the bad state disproportionately affects the United States. In a complete markets setting, one can derive the following:

$$\text{Risk premium on variance} = \mathbb{E}^{\mathbb{P}}_t(\{-\frac{2}{\tau}\}\log(\frac{S^{\text{us|j}}_{t+\tau}}{F^{\text{us|j}}_{t,\tau}})) - \mathbb{E}^{\mathbb{Q}}_t(\{-\frac{2}{\tau}\}\log(\frac{S^{\text{us|j}}_{t+\tau}}{F^{\text{us|j}}_{t,\tau}})) \qquad (2)$$

$$= -\frac{1}{2}\Psi_{\text{us}} \underbrace{(1+\hbar-\mho(1-\hbar))}_{>0} \underbrace{\left(\log(\frac{1-\Psi_{\text{j}}}{1-\Psi_{\text{us}}}) + \log(\frac{1+\Psi_{\text{us}}}{1+\Psi_{\text{j}}})\right)}_{>\,0 \text{ when } \Psi_{\text{us}}>\Psi_{\text{j}}} < 0. \qquad (3)$$

Our use of the payoff $\{-\frac{2}{\tau}\}\log(\frac{S^{\text{us|j}}_{t+\tau}}{F^{\text{us|j}}_{t,\tau}})$ is in analogy to the dollar/euro currency VIX (ticker: EVZ). The workings of this model further imply the following:

$$\text{Risk premium on asymmetry} = \mathbb{E}^{\mathbb{P}}_t(\{\log(\frac{S^{\text{us|j}}_{t+\tau}}{F^{\text{us|j}}_{t,\tau}})\}^3) - \mathbb{E}^{\mathbb{Q}}_t(\{\log(\frac{S^{\text{us|j}}_{t+\tau}}{F^{\text{us|j}}_{t,\tau}})\}^3) \qquad (4)$$

$$= \frac{1}{4}\Psi_{\text{us}}(1+\hbar-\mho(1-\hbar))\left(\log^3(\frac{1-\Psi_{\text{j}}}{1-\Psi_{\text{us}}}) + \log^3(\frac{1+\Psi_{\text{us}}}{1+\Psi_{\text{j}}})\right) > 0. \qquad (5)$$

The implication is that the return distribution under $\mathbb{Q}$ in this model is more left skewed than under $\mathbb{P}$.

This two-currency economy encapsulates a number of features. Notably, the currency risk premium, defined as $\mathbb{E}_t^{\mathbb{P}}(\frac{S_{t+\tau}^{\text{us}|\text{j}}}{F_{t,\tau}^{\text{us}|\text{j}}})-1$, is $\frac{\Psi_{\text{us}}(\Psi_{\text{us}}-\Psi_{\text{j}})(1+\hbar-\mho(1-\hbar))}{2(1-\Psi_{\text{us}}^2)}>0$. The currency with a positive risk premium is viewed as being risky from the perspective of the United States. Moreover, a higher exposure to the tail (that is, $\hbar$) induces a higher currency risk premium, and the U.S. dollar appreciates in the unpleasant state.

Our framework — when viewed in conjunction with Results 1 through 3 (details in section 4) — provides a perspective on the following hypotheses about risk premiums on out-of-the-money (OTM) options:

Hypothesis 1 (risk premiums of currency puts). If the single-name currency risk premium is positive, then the put risk premium on the single-name currency can be negative.

Hypothesis 2 (risk premiums of currency calls). The risk premiums of currency calls can switch sign in the cross-section of currencies.

Hypothesis 3 (currency volatility risk premium). The single-name currency volatility risk premium can be negative or positive.

The notion of macroeconomic disparities across economies is conceptually crucial to our empirical framework and to model building. One may envision that the United States is endowed with relatively healthier institutions, including a competent central bank and less compromised monetary policies, and offers more protection for investors. These institutional features may be at the root of the stature of the U.S. dollar as the king of all currencies.

The role of the U.S. dollar is multifaceted, which is reflected in our hypotheses for option premiums. For example, the U.S. dollar is a preferred reserve currency (held in significant quantity by central banks). Investors trust that the U.S. dollar will retain its value, in essence, offering safe haven during periods of heightened economic uncertainties. Additionally, the U.S. dollar is used to conduct international trade and financial transactions. Our hypothesis about call premiums is consistent with a possible lackluster need to insure against sharp U.S. dollar depreciations, as reflected in call prices. The exception to this behavior could be time-sensitive dollar revenue exposures of economies with exports invoiced in U.S. dollars.

Global investors are also apprehensive about potential devaluations in the foreign currencies. In comparison with the U.S. dollar, foreign currencies may be riskier. Investors recognize, for example, that during bad economic times, the foreign currency can depreciate with respect to the U.S. dollar. The workings of flight to quality can further exacerbate declines in a foreign currency. Our hypothesis about negative put risk premiums is consistent with downside protection motives in the currency markets.

Finally, unlike equities, the leverage effect in currencies is presumably weaker. In other words, a fall in the currency value is *not* accompanied by a substantial rise in volatility that potentially makes options more expensive through the vega. If investors are committed to holding the U.S. dollar, then the risk premium on currency volatility can be anticipated to be small in magnitude and can be negative or positive.

2.2. Daily options data on 15 single-name currencies with the U.S. dollar as the base currency

Our data consist of *daily* quotes on single-name currency option prices as well as observations on forward and spot exchange rates. By convention, currency option prices are quoted in the form of 10-delta, 25-delta, and at-the-money (ATM) put or call volatilities. Importantly, the positions in currency options and forwards can be *initiated each day* and settle in 30 days.

Table 1 provides the list of single-name currency options, the source for which is a major bank. Our quote convention is such that the base currency is the U.S. dollar and the reference is the foreign currency. In our daily data, the earliest start date is 1/3/2000, and the end date for all options is 10/31/2019.

Our empirical analysis is based on the following 15 currency option pairs (with the U.S. dollar as the base): Australian dollar, Canadian dollar, Czech koruna, Danish krone, euro, Hungarian forint, Japanese yen, South Korean won, New Zealand dollar, Norwegian krone, Polish zloty, South African rand, Swedish krona, Swiss franc, and the British pound. Because the currency option contracts unravel in 30 days, there are 73,473 option expiration cycle observations in our sample.

Our sample includes the G10 currencies, the world's most actively traded currencies, which account for the majority of turnover in currency markets.[3] We complement this sample by including six non-G10

[3] According to the 2019 survey of turnover in foreign exchange markets (compiled by the Bank for International Settlements), these currencies plus the dollar account for more than 85% of the total global turnover of over-the-counter instruments.

pairs. Whereas most currency options research relies on data at the monthly frequency, the use of options at the daily frequency, while maintaining a constant maturity of 30 days, is a unique feature of our study.[4]

We employ the following notation:

τ: Remaining days to maturity of the forward and option contracts, set equal to 30/365.

$S_t^{\mathrm{us|j}}$: Spot price (at day t) of one unit of currency j in terms of the U.S. dollar. Henceforth, a rise in $S_t^{\mathrm{us|j}}$ is associated with the appreciation (depreciation) of the reference (base) currency.

$F_{t,\tau}^{\mathrm{us|j}}$: Forward price of one unit of currency j in terms of the U.S. dollar with settlement in τ days from t.

r_t^{us} (r_t^{j}): Interest-rate on the U.S. dollar (foreign currency) deposit for τ-day horizons (known at day t).

$\triangle_p$ ($\triangle_c$): Delta of a put (call) on the currency (that is, ATM, 25, or 10 delta).

$\sigma_t[\triangle_p]$: Volatility quote of a put on day t, where $\triangle_p$ take values ATM, 25, or 10 (deepest OTM put).

$\sigma_t[\triangle_c]$: Volatility quote of a call on day t, where $\triangle_c$ take values ATM, 25, or 10 (deepest OTM call).

Let K_{ATM}, $K_{\triangle_p}$, and $K_{\triangle_c}$ be the strike prices corresponding to the respective option deltas, and $\mathcal{N}^{-1}[.]$ represent the inverse of the standard normal cumulative distribution. We apply the following conversion formulas (see, for example, Wystup (2006)) to obtain strike prices for the quoted volatilities:

$$K_{\mathrm{ATM}} = F_{t,\tau}^{\mathrm{us|j}} \exp(\frac{1}{2}\sigma_t^2[\mathrm{ATM}]\,\tau), \tag{6}$$

$$K_{\triangle_p} = F_{t,\tau}^{\mathrm{us|j}} \exp(\frac{1}{2}\sigma_t^2[\triangle_p]\,\tau + \sigma_t[\triangle_p]\,\sqrt{\tau}\,\mathcal{N}^{-1}[-\exp(r_t^{\mathrm{j}}\,\tau)\,\triangle_p]), \quad \text{and} \tag{7}$$

$$K_{\triangle_c} = F_{t,\tau}^{\mathrm{us|j}} \exp(\frac{1}{2}\sigma_t^2[\triangle_c]\,\tau - \sigma_t[\triangle_c]\,\sqrt{\tau}\,\mathcal{N}^{-1}[\exp(r_t^{\mathrm{j}}\tau)\,\triangle_c]). \tag{8}$$

With the strike prices of the currency options (as in equations (6)–(8)) and quoted volatilities, we then calculate the corresponding put and call prices using the Garman and Kohlhagen (1983) formula (e.g., Wystup (2006)). The put (respectively, call) price with strike price K and remaining maturity τ is denoted

[4] While the literature on currencies is vast, the number of studies that exploit currency options data across available strikes and single names is much smaller. Our paper joins, among others, Bates (1996); Carr and Wu (2007); Bakshi, Carr, and Wu (2008); Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011); Jurek (2014); Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2013); Della Corte, Sarno, and Ramadorai (2016); Londono and Zhou (2017); Daniel, Hodrick, and Lu (2017); and Della Corte, Kozhan, and Neuberger (2021).

as $\text{put}_t^{\text{us}|\text{j}}[K]$ (respectively, $\text{call}_t^{\text{us}|\text{j}}[K]$). We omit the τ dependence on the currency option prices for brevity.[5]

We compute risk reversals as the volatility of a put minus that of call for a fixed delta. Table 1 indicates considerable heterogeneity in risk-neutral currency return distributions, with non-G10 pairs displaying higher volatilities and more pronounced risk reversals. The average risk reversals are negative only for the Japanese yen and, to a lesser degree, the Swiss franc, two traditional funding or safe-haven currencies.

3. Facts about currency option risk premiums

The implications for option risk premiums are shaped by the feature that the U.S. dollar plays a critical role in the global financial system. The goal of this section is to empirically examine these implications and investigate if, and how, macroeconomic disparities between the United States and other economies differentially affect downside versus upside currency option risk premiums.

We answer the following questions: What are the quantitative features of single-name currency option premiums? Is there a dichotomy between option premiums for currency baskets obtained by dynamically sorting the currency universe based on interest-rate differentials (with the U.S. dollar as the base currency)? In other words, are the option premiums for investment and funding currencies different? How do these option premiums change in response to heightened economic uncertainties? Finally, which economy-pair characteristics describe the heterogeneity in option risk premiums across currencies?

3.1. Put risk premiums for currencies are overwhelmingly negative

Each day in our sample, we buy a 10-delta (or 25-delta) put on a currency and compute the excess returns from holding this option over the subsequent 30 days, as follows:

$$\mathbf{z}^{\text{put}}_{\{t\to t+\tau\}}[K_{10\triangle_p}] \;=\; \frac{\max(K_{10\triangle_p} - S^{\text{us}|\text{j}}_{t+\tau}, 0)}{\text{put}_t[K_{10\triangle_p}]} \;-\; \exp(r^{\text{us}}_t\,\tau), \tag{9}$$

[5] Aiding interpretations, the OTM put, say, with strike $K_d < S_t$, provides a payoff if the reference currency (respectively, the U.S. dollar) *depreciates* (respectively, appreciates) beyond the threshold of K_d. In contrast, the OTM call, say, with strike $K_u > S_t$, provides a payoff if the reference currency (respectively, the U.S. dollar) *appreciates* (respectively, depreciates) beyond K_u. As we show, our empirical analysis and theoretical explorations are aided by considerable variation in state-contingent depreciations and appreciations (i.e., option returns) across single-name currencies.

where $K_{10\triangle_p}$ corresponds to the strike price of a 10 delta put, $S_{t+\tau}^{\text{us}|\text{j}}$ is the (utilized) settlement price of the foreign currency, and $\text{put}_t[K_{10\triangle_p}]$ is the put price. We proxy r_t^{us} by the 30-day U.S. Treasury bill rate.

The $\mathbb{P}$-measure expectation of $\mathbf{z}_{\{t\to t+\tau\}}^{\text{put}}[K_{10\triangle_p}]$ conditional on filtration $\mathcal{F}_t$, that is, $\mathbb{E}_t^{\mathbb{P}}(\mathbf{z}_{\{t\to t+\tau\}}^{\text{put}}[K_{10\triangle_p}])$, defines the put risk premium. If excess returns of puts are aligned with the ex ante expected excess return of puts and are negative, then market participants are paying a premium to protect against downward movements in currencies. This effect is equivalent to the appreciation of the U.S. dollar.

Table 2 shows that, among the G10 currency pairs, the 10 delta put moneyness ranges between 3.5% for the Canadian dollar and 5.1% for the New Zealand dollar. Our rationale for keeping the option delta fixed is to account for differences in currency volatility across names and in the time-series.

The salient aspect of table 2 (panel A) is that the mean return to holding 10 delta puts is negative for *14 of the 15* pairs. These put risk premiums are statistically significant for six out of the nine G10 pairs, and the evidence is stronger for non-G10 pairs, all of which display negative and significant put risk premiums. In our exercises, we evaluate statistical significance using lower and upper 95% bootstrap confidence intervals for average put option excess returns.[6] We mark in bold the put risk premiums that are statistically significant.

Inspecting the results from equal-weighted currency baskets, we find that the put risk premiums are −23.4% over a holding period of 30 days (not annualized) for G10 pairs and −24.8% among the 15 pairs. Implying the presence of greater insurance concerns, the put risk premiums are higher for non-G10 pairs (−30.9%).

Akin to 10-delta puts, 25-delta puts (panel B) also manifest negative premiums, but the distinction is that they are less negative. We examine bootstrap confidence intervals for the differences in the adjacent strikes and find that they do not overlap. The feature that put risk premiums are smaller in magnitude for deeper OTM puts (that is, *more negative* for most currencies or *less positive* for Japan) agrees with our theory.

[6] In terms of the reliability of our assessments for the significance of option premiums, our evidence is based on 44,408 (respectively, 73,473) option expiration cycles for the G10 (respectively, all 15 currencies).

All in all, table 2 provides support for the notion that the U.S. dollar is the king of all currencies. Our evidence comes in the form of buying insurance against the depreciation of foreign currencies.

3.2. Call risk premiums are both positive and negative across currency pairs

In analogy to puts, we buy a 10-delta (or 25-delta) call each day and compute the excess returns over the subsequent 30 days, as follows:

$$\mathbf{z}^{\text{call}}_{\{t\to t+\tau\}}[K_{10\triangle_c}] \;=\; \frac{\max(S^{\text{us}|\text{j}}_{t+\tau} - K_{10\triangle_c}, 0)}{\text{call}_t[K_{10\triangle_c}]} \;-\; \exp(r^{\text{us}}_t \tau), \tag{10}$$

where $K_{10\triangle_c}$ is the strike of a 10 delta call and $\text{call}_t[K_{10\triangle_c}]$ denotes the call price.

Comparing the results in table 3 for call premiums with those for put risk premiums in table 2, we can emphasize several findings. First, for the farther OTM options (that is, 10 delta), seven (respectively, three) currency pairs have negative (respectively, positive) and significant call premiums. In line with our theory, which attributes a role to the sign of single-name currency risk premiums, the call premiums are both negative and positive. Call premiums are typically higher, at 25 delta versus 10 delta, but three currencies with positive and significant call premiums offer a noticeable contrast. Thus, the heterogeneity in call premiums not only arises from their signs but also from not being always decreasing in strike.

Second, focusing on G10 currencies, we find that the Japanese yen (-57.3%), the Canadian dollar (-27.8%), the U.K. pound (-22.5%), and the euro (-19.8%) display reliably negative and significant call premiums. Our results suggest that negative call premiums are concentrated among economies that are heavy exporters to the United States.[7] One potential driver of negative call premiums is then sizable dollar revenue exposures, which is a form of U.S. prominence in global trading arrangements (see, e.g., Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Moller (2020)). Accordingly, our evidence is aligned with hedging U.S. dollar depreciations by an economically relevant set of economies.

[7] According to the IMF Direction of Trade Statistics, for December of 2019, 12.2%, 10.6%, 7.7%, and 6.3% of the total exports of Japan, Canada, the euro area, and the United Kingdom, respectively, are to the United States.

The magnitude and negative nature of call premiums may also be connected to the use of funding currencies, for example, the Japanese yen, in carry trade strategies. If the Japanese yen is sold forward or shorted in the futures market (because it is mostly in contango), then, buying calls offers protection against an appreciating yen. In section 3.4, we explore this mechanism by buying currency calls aligned with a dynamically rebalanced set of currencies with relatively low interest-rates.

Overall, our evidence supports the view that risk premiums on calls are, on average, much smaller relative to puts. Fixing delta to 10, we find that the call risk premium for the 15 currency basket is -3.4%, as opposed to -24.8% for puts. The estimate of this difference between put and call premiums is -21.4%, with a lower and upper bootstrap confidence interval of -0.31 and -0.12, respectively. The main takeaway is that markets worry more about downside movements in foreign currencies than about movements on the upside. Broadly speaking, they dislike more the states in which the U.S. dollar is appreciating.[8]

3.3. *Currency volatility risk premiums are not uniformly negative and are near zero for baskets*

The nature of the volatility risk premiums can be extracted from the excess return of a currency straddle, calculated as follows:

$$\mathbf{z}^{\text{straddle}}_{\{t\to t+\tau\}} = \frac{\max(S_t^{\text{us}|\text{j}} - S_{t+\tau}^{\text{us}|\text{j}}, 0) \ + \ \max(S_{t+\tau}^{\text{us}|\text{j}} - S_t^{\text{us}|\text{j}}, 0)}{\text{put}_t[S_t^{\text{us}|\text{j}}] \ + \ \text{call}_t[S_t^{\text{us}|\text{j}}]} \ - \ \exp(r_t^{\text{us}}\,\tau). \tag{11}$$

We observe from table 4 that the Canadian dollar, the Japanese yen, the Danish krone, the South Korean won, and the South African rand have *negative* and significant straddle excess returns. However, the associated magnitudes are relatively small and range between -2.3% to -5.4%. In contrast, the Australian dollar and the Czech koruna are distinguished by a *positive* and significant volatility risk premium.

The robust finding in table 4 is that single-name currencies elicit small negative, small positive, or indistinguishable from zero volatility risk premiums. Notably, the volatility risk premium vanishes in

[8] Inviting additional comparisons, there are asymmetries in currency option returns to the downside (that is, dollar appreciations) versus the upside (that is, dollar depreciations). Specifically, the largest return realization of puts (across all expiration cycles) is more pronounced in comparison with calls. This data attribute is noteworthy because the proportion of the option expiration cycles that yields positive returns is comparable across the puts and calls (see the columns labeled $\mathbb{1}_{\{\mathbf{z}>0\}}$).

currency baskets: It is -0.7% (respectively -0.5) per month for G10 (respectively, non-G10) pairs, and the average across all 15 pairs is -0.1%; in all cases, the 95% bootstrap confidence intervals bracket zero. Our results suggest that currency volatility does not appear to be a big concern to markets, and this indifference may be a consequence of the U.S. dollar being at the center of the international financial system.

3.4. Disparities in option risk premiums when currencies are dynamically sorted on $\log(\frac{F_t^{\text{us}|\text{j}}}{S_t^{\text{us}|\text{j}}})$

The argument we framed about the U.S. dollar being the king of all currencies is that markets will be more apprehensive about foreign currency depreciations. Therefore, downward movements in the foreign currency would be hedged by buying OTM currency puts. If, as in the traditional carry trade, investors buy investment currencies (those with high $r_t^{\text{j}} - r_t^{\text{us}}$) and possibly cover their downside risk, would this feature imply additional pressure on negative put risk premiums? Isomorphically, if investors engage in selling funding currencies (those with low $r_t^{\text{j}} - r_t^{\text{us}}$) and possibly cover their positions by buying OTM currency calls, would it pressure negative call premiums?

To address these questions, we dynamically rank currency pairs by their $\log(\frac{F_t^{\text{us}|\text{j}}}{S_t^{\text{us}|\text{j}}})$ on day t (see, e.g., Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and Lustig, Roussanov, and Verdelhan (2011)). The five currencies with the *lowest* $\log(\frac{F_t^{\text{us}|\text{j}}}{S_t^{\text{us}|\text{j}}})$ are classified as having the *highest* interest-rate differentials. In contrast, the five currencies with the highest $\log(\frac{F_t^{\text{us}|\text{j}}}{S_t^{\text{us}|\text{j}}})$ are classified as having the lowest interest-rate differentials. Thus, we divide our currency universe into three bins, with the extreme ends constituting the set of investment and funding currencies, respectively. Next, we buy the associated 10 delta puts, 10 delta calls, and straddles. We then compute the equal-weighted excess returns of the option positions over the subsequent 30 days. The three currency sets are dynamically re-balanced each day.

Table 5 presents four noteworthy patterns. First, the carry strategy (panel B) earns, on average, 3.8%, and the bootstrap confidence intervals do not bracket zero. In essence, our option risk premium patterns are anchored to the empirical observation that the long-leg (respectively, short-leg) of the carry is, on average, profitable (respectively, unprofitable).

Second, put risk premiums on investment and funding currencies, while both negative, are more negative for investment currencies. The disparity in put risk premium is -9% per month and is statistically significant. Thus, even though foreign currencies garner downside hedging interest, our analysis reveals that an incremental negative risk premium is attached to the investment currencies.

Third, the risk premium differential on 10 delta calls is equally revealing. Specifically, the call risk premium for the basket of funding currencies is -11.1%, and it is statistically more negative than that for investment currencies — in this case, the disparity in risk premiums is 9%. Such an observation ties into our reasoning that negative call premiums arise partly because of the tendency to hedge against the appreciation of funding currencies in the carry trade. Both features can be traced to the dominance of the United States and to the preponderance of the U.S. dollar in global financial and trade transactions.

Fourth, the volatility risk premiums are small and statistically indistinguishable between the investment and funding currencies. The risk premium differential is -0.9%, with a bootstrap confidence interval of -2.3% and 0.5%.

3.5. Option risk premiums and heightened economic uncertainty

The U.S. dollar is conceivably underpinned by sound macroeconomic fundamentals. This view has been shaped by, among others, the studies of Gourinchas and Rey (2007), Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Rull (2009), and Farhi and Maggiori (2018). These authors have emphasized the safe-haven qualities of the U.S. dollar during periods of heightened economic uncertainty.

In our exercise, economic uncertainty is characterized using VIX (volatility) states. Specifically, heightened economy uncertainty is measured by realizations of $\text{VIX}_t \geq 40$. For comparison, we divide our daily sample into five VIX bins: (i) $\text{VIX}_t < 11$ (count of 241), (ii) $11 \leq \text{VIX}_t < 17$ (count of 2,270), (iii) $17 \leq \text{VIX}_t < 24$ (count of 1,615), (iv) $24 \leq \text{VIX}_t < 40$ (count of 1,102), and (v) $\text{VIX}_t \geq 40$ (count of 189). The highest VIX states compose 3% of our sample, and these states are often marked by retreating equity markets, the withdrawal of international credit flows, and the migration to safer assets.

Table 6 shows the risk premiums on options for each bin. These entries are to be interpreted as partitioned average 30-day excess returns that are conditional on filtration $\mathcal{F}_t = \text{VIX}_t$. In other words, we take a position in currency options conditional on $\mathcal{F}_t$ and compute the subsequent excess returns.

Put premiums are negative in the high VIX states for both the investment and funding currencies. While buying puts on currencies can deliver big subsequent payoffs in high volatility states, the cost of protection against the appreciation of the U.S. dollar rises simultaneously. We find that put premiums allied to high volatility states are indistinguishable between investment and funding currencies.[9]

The VIX responds to bad information, and our findings for 10 delta calls offer further separation on currencies with low $r_t^{\text{j}} - r_t^{\text{us}}$. When $\text{VIX}_t \geq 40$, the distinction to draw is that the funding currencies do not sufficiently appreciate against the U.S. dollar to yield positive call premiums. In this state, the negative call risk premium on the funding currencies drives the positive and significant difference in the call premiums for investment versus funding currencies. This difference in premiums remains positive when $24 \leq \text{VIX}_t < 40$, although, in these states, the calls on both investment and funding currencies are profitable but significant only for investment currencies. Our analysis implies that investment currencies experience a rebound in the subsequent 30 days, consistent with call premiums that are positive and significant.

Currencies appear exposed to volatility in the low VIX states, as reflected in the profitability of straddles. Many more symmetric moves in the return tails are implied, more so for the funding currencies. Specifically, when $\text{VIX}_t < 11$, the difference in volatility risk premium between investment and funding currencies is -19%, implying that the volatility risk premium is reliably more positive for funding currencies. Funding currencies are more prone to experiencing large moves against the U.S. dollar following low VIX states but not following high VIX states, as evidenced by the positive straddle risk premium differential.

The essence of our evidence is that the risk premiums on puts, calls, and straddles of investment cur-

[9] The distinction in the riskiness of investment and funding currencies is more evident in the put risk premium differentials when $24 \leq \text{VIX}_t < 40$. This volatility bin is allied to more pronounced downward movements in the investment currencies, leading to a positive and statistically significant put premium differential between investment and funding currencies. Although put risk premiums remain robustly negative in the lower-tier volatility states ($\text{VIX}_t < 24$), they are more negative for investment currencies.

rencies exceed those of funding currencies during high volatility states. Influenced by the prominence of the U.S. dollar, investors are willing to accept small negative expected option returns for funding currencies in the bad economic states. This evidence is backed by the evidence on carry trades, which are associated with the highest average returns (8.6%) in the high VIX states. This outcome is driven by a subsequent rise in the (risky) investment currencies, in combination with positive returns to the short leg of the carry.

3.6. Option risk premiums and global currency volatility

The evidence in Menkhoff, Sarno, Schmeling, and Schrimpf (2012) shows that high interest-rate currencies tend to provide low returns when global currency volatility is unexpectedly high. Mancini, Ranaldo, and Wrampelmeyer (2013) analyze the effect of liquidity risk on carry trades and furnish evidence that funding (respectively, investment) currencies offer insurance against (respectively, exposure to) liquidity risk. One may envision that these risks are likely to be pronounced in high currency volatility states.

Following the intuition and the evidence in the literature for the relevance of currency volatility for currency strategies, we calculate a measure of global currency volatility. To do so, we first build the time-series of single-name option-based volatility, which we label *currency VIX*, as follows (with $\tau = 30/365$):

$$\underbrace{\mathrm{VIX}_t^{\mathrm{fx,j}}}_{\text{currency VIX}} \equiv \mathbb{E}_t^{\mathbb{Q}}(\{-\frac{2}{\tau}\}\log(\frac{S_{t+\tau}^{\mathrm{us|j}}}{F_{t,\tau}^{\mathrm{us|j}}})) = \frac{2e^{r_t^{\mathrm{us}}\tau}}{\tau}\int_0^{F_{t,\tau}^{\mathrm{us|j}}} \frac{1}{K^2}\,\mathrm{put}_t[K]\,dK + \frac{2e^{r_t^{\mathrm{us}}\tau}}{\tau}\int_{F_{t,\tau}^{\mathrm{us|j}}}^{\infty} \frac{1}{K^2}\,\mathrm{call}_t[K]\,dK. \quad (12)$$

Then, at each t, we equal-weight single name $\mathrm{VIX}_t^{\mathrm{fx,j}}$ and label it as $\overline{\mathrm{VIX}}_t^{\mathrm{fx}}$. Table 10 (panel B) indicates that the extracted first principal component of currency VIX has an almost equal loading on its constituents.

The coverage of the global currency volatility states is as follows: (i) $\overline{\mathrm{VIX}}_t^{\mathrm{fx}} < 8$ (count of 241), (ii) $8 \leq \overline{\mathrm{VIX}}_t^{\mathrm{fx}} < 10$ (count of 1,082), (iii) $10 \leq \overline{\mathrm{VIX}}_t^{\mathrm{fx}} < 12$ (count of 1,335), (iv) $12 \leq \overline{\mathrm{VIX}}_t^{\mathrm{fx}} < 16$ (count of 1,332), and (v) $\overline{\mathrm{VIX}}_t \geq 16$ (count of 451). The correlation between the equity VIX_t and $\overline{\mathrm{VIX}}_t^{\mathrm{fx}}$ is 0.80.

Entries in table 7 are partitioned average excess returns, conditional on filtration $\mathcal{F}_t = \overline{\mathrm{VIX}}_t^{\mathrm{fx}}$. We find that the put risk premium differential between investment and funding currencies is positive and significant when $\overline{\text{VIX}}_t \geq 16$. Thus, our evidence suggests that, conditional on being in a high $\overline{\text{VIX}}_t^{\text{fx}}$ state, the cost of protection rises disproportionately for funding currencies.

Call premiums are negative and significant for investment currencies and more so for funding currencies in states when $\overline{\text{VIX}}_t^{\text{fx}}$ is high. However, there is no discernible difference in call premiums between investment and funding currencies. In these states, the straddle risk premium differential is positive and significant, a result driven by the lack of profitability of straddles for the funding currencies.

Reinforcing our observations about option premiums, the long leg of the carry trade is profitable when initiated in the high $\overline{\text{VIX}}_t^{\text{fx}}$ states, with an average return of 5.6% annualized. The documented positive premiums on the put option of investment currencies simultaneously with the long leg of the carry are not puzzling when reconciled through the lens of the convexity of the option payoffs.

Akin to the evidence from the high VIX states, the risk premiums on puts, calls, and straddles of investment currencies *exceed* those of funding currencies in the high $\overline{\text{VIX}}_t^{\text{fx}}$ states, although this difference is not statistically significant for calls. These option risk premiums patterns connect to economic phenomena in which the U.S. dollar wields sway as a global safe asset.

3.7. Panel regressions linking option premiums to macroeconomic disparities

To investigate how macroeconomic disparities between the United States and other economies affect option premiums, we utilize a panel regression framework. For example, for put risk premiums, we estimate the following regression with nonoverlapping monthly observations:

$$\begin{aligned}\underbrace{\mathbf{z}^{\text{put,j}}_{\{t\to t+\tau\}}}_{\text{next 30 days}} &= \delta_0 + \delta_1 (r_t^{\text{j,5year}} - r_t^{\text{us,5year}}) + \delta_2 \text{QV}_t^{\text{fx,j}} + \delta_3 \text{MA30}_t^{\text{fx,j}} \\ &\quad + \delta_4 \text{RR10}_t^{\text{j}} + e^{\text{j}}_{\{t,t+\tau\}} \;\text{ for j} = 1,\ldots,15 \;\text{ and }\; t = 1,\ldots,213. \qquad (13)\end{aligned}$$

Underscoring distinctions across economies, in table 8, we consider disparity variables known at time t, namely, the interest-rate differentials on five-year government bonds ($r_t^{\text{j,5year}} - r_t^{\text{us,5year}}$), the quadratic variation of currency returns ($\text{QV}_t^{\text{fx,j}}$), the currency excess returns over a 30-day trailing window ($\text{MA30}_t^{\text{fx,j}}$), and the 10 delta risk reversals (RR10_t^{j}). Our choice of variables can be motivated on empirical and/or intuitive grounds. The variable construction is described in the note to table 8.

We consider both currency fixed effects and year fixed effects in the panel regressions. Our rationale for the currency fixed effect is to account for time-invariant currency characteristics (for example, distance and the centrality of the United States). Allowing for year fixed effects internalizes the impact of global risks on the cross-section of option premiums. We consider robust standard errors that are clustered by currency. Parameter significance implies a relationship in both the cross-section and time-series.

Corroborating the ability of macroeconomic disparities to capture currency option premiums, we emphasize four findings. First, there is a positive association between 10 delta put risk premiums and $r^{\text{j,5year}} - r^{\text{us,5year}}$. Higher interest-rate differentials also forecast the appreciation of these economies' currencies against the U.S. dollar. Second, the dimension of $\text{QV}^{\text{fx,j}}$ is negatively and significantly related to all three option premiums, which suggests that higher currency volatility is related to increasing concerns against currency changes, irrespective of the direction. Third, the negative coefficient on $\text{MA30}^{\text{fx,j}}$ for puts conveys that downside protection concerns are more pronounced for economy pairs with previous currency appreciations against the U.S. dollar. Finally, risk reversals manifest riskiness in the sense of Farhi and Gabaix (2016). Specifically, we assess whether higher RR10^{j}s forecast higher currency risk premiums and more negative put risk premiums. Although the association of RR10^{j} with put and currency risk premiums is as hypothesized, the highest (absolute) t-statistic is 1.66.

Overall, our panel regression framework suggests that differences in macroeconomic outcomes, especially the quadratic variation in currency returns, help to forecast option risk premiums.

4. A theory of option risk premiums anchored to the empirical findings

This section presents a tractable model of currency dynamics and explores its consistency with the empirical properties of currency option risk premiums. Our setup inherits several features of the model in section 2 but remedies the absence of dynamically evolving global risks. Using this framework, we can accommodate differential sensitivity of economies to different candidates for global risks, namely, those arising from volatility, disaster probabilities, and tail events.

We are interested in theoretically decomposing currency option risk premiums and then highlighting the potential economic mechanisms for the documented empirical patterns. Our primary question is the following: Can this model reproduce the properties of currency option risk premiums?

4.1. Motivation for the theoretical elements

Three empirical findings motivate the model:

1. *Currency risk premiums are of variable sign.* Table 9 shows that 12 of the 15 currencies display positive currency risk premiums, and 10 of these 12 pairs are positive and significant. The Japanese yen offers a deviation from this pattern and displays a significantly negative currency risk premium.
2. *Option risk premiums have at least two predominant drivers.* Table 10 indicates that two principal components together capture 59%, 51%, and 55% of the variance in the single-name excess returns of 10 delta puts, 10 delta calls, and straddles, respectively.
3. *There are large down and up tail movements.* As depicted by $\mathbb{1}_{\{\mathbf{z}>0\}}$, 10 delta puts have positive excess returns in 6% to 10% of the cycles (table 2). Correspondingly, 10 delta calls have positive excess returns in 4% to 10% of the cycles (table 3). A significant number of directional down and up movements are implied by the $\mathbb{1}_{\{\mathbf{z}>0\}}$ statistics for currency straddles (table 4).

The dynamics of the domestic pricing kernel (denoted by M_t^{us}) and the foreign pricing kernel (denoted by M_t^{j}) are equipped with jumps of random amplitude and stochastic volatility, as follows:

$$\frac{dM_t^{\text{us}}}{M_{t-}^{\text{us}}} = -r_t^{\text{us}}\,dt + \beta_{\text{us}}\sqrt{\mathbf{v}_t}\,d\mathbb{W}_t^{\mathbb{P},\mathbf{v}} + \eta_{\text{us}}\sqrt{\mathbf{b}_t}\,d\mathbb{W}_t^{\mathbb{P},\mathbf{b}} + (e^{\alpha_{\text{us}}x}-1)\,d\mathbb{N}_t - \mathbf{b}_t\,\mathbb{E}_t^{\mathbb{P}}(e^{\alpha_{\text{us}}x}-1)dt \quad (14)$$

$$\frac{dM_t^{\text{j}}}{M_{t-}^{\text{j}}} = -r_t^{\text{j}}\,dt + \beta_{\text{j}}\sqrt{\mathbf{v}_t}\,d\mathbb{W}_t^{\mathbb{P},\mathbf{v}} + \eta_{\text{j}}\sqrt{\mathbf{b}_t}\,d\mathbb{W}_t^{\mathbb{P},\mathbf{b}} + \underbrace{(e^{\alpha_{\text{j}}x}-1)\,d\mathbb{N}_t}_{\text{jump contribution}} - \underbrace{\mathbf{b}_t\,\mathbb{E}_t^{\mathbb{P}}(e^{\alpha_{\text{j}}x}-1)dt}_{\text{compensator}}, \quad (15)$$

where r_t^{us} and r_t^{j} are (deterministic) interest rates in the United States and the foreign economy, respectively. Furthermore, $\mathbb{W}_t^{\mathbb{P},\mathbf{v}}$ and $\mathbb{W}_t^{\mathbb{P},\mathbf{b}}$ are standard Brownian motions that affect the evolution of the pricing kernels.

Jumps in the pricing kernels are modeled as arriving at random times t_ℓ with jump intensity $\mathbf{b}_t$ (which is an autonomous process). The effect of jumps is asymmetric with $M_{t_\ell}^{\text{us}} = M_{t_{\ell-}}^{\text{us}}\, e^{\alpha_{\text{us}} x_\ell}$ and $M_{t_\ell}^{\text{j}} = M_{t_{\ell-}}^{\text{j}}\, e^{\alpha_{\text{j}} x_\ell}$. The size of the jump x (disaster) is an *i.i.d* random variable and is independent of the Poisson process $\mathbb{N}_t$.

All sources of uncertainty (that is, $\mathbb{W}_t^{\mathbb{P},\mathbf{v}}$, $\mathbb{W}_t^{\mathbb{P},\mathbf{b}}$, $\mathbb{N}_t$, x) are uncorrelated. We assume

$$\text{Diffusive (component) variance under } \mathbb{P}: \quad d\mathbf{v}_t = \kappa_{\mathbf{v}}^{\mathbb{P}}\Big(\frac{\theta_{\mathbf{v}}^{\mathbb{P}}}{\kappa_{\mathbf{v}}^{\mathbb{P}}} - \mathbf{v}_t\Big)\, dt \,+\, \sigma_{\mathbf{v}}\sqrt{\mathbf{v}_t}\, d\mathbb{W}_t^{\mathbb{P},\mathbf{v}} \tag{16}$$

$$\text{Disaster probability under } \mathbb{P}: \quad d\mathbf{b}_t = \kappa_{\mathbf{b}}^{\mathbb{P}}\Big(\frac{\theta_{\mathbf{b}}^{\mathbb{P}}}{\kappa_{\mathbf{b}}^{\mathbb{P}}} - \mathbf{b}_t\Big)\, dt \,+\, \sigma_{\mathbf{b}}\sqrt{\mathbf{b}_t}\, d\mathbb{W}_t^{\mathbb{P},\mathbf{b}} \tag{17}$$

$$\text{Poisson jump in the pricing kernels}: \quad d\mathbb{N}_t = \begin{cases} 1 & \text{with probability } \mathbf{b}_t\, dt \\ 0 & \text{with probability } 1 - \mathbf{b}_t\, dt \end{cases} \tag{18}$$

$$\text{Disasters}: \quad x \sim \text{i.i.d. Normal}(\mu_x, \sigma_x^2). \tag{19}$$

The compensator $\mathbf{b}_t\, \mathbb{E}_t^{\mathbb{P}}(e^{\alpha_{\text{j}} x} - 1)\, dt$ ensures that the drift rate of $\frac{dM_t^{\text{j}}}{M_t^{\text{j}}}$ is $-r_t^{\text{j}}$.

A notable aspect is that the sensitivity of $\log(\frac{M_{t+\tau}^{\text{j}}}{M_{t-}^{\text{j}}})$ to $\int_t^{t+\tau}\sqrt{\mathbf{v}_u}\, d\mathbb{W}_u^{\mathbb{P},\mathbf{v}}$ and $\int_t^{t+\tau}\sqrt{\mathbf{b}_u}\, d\mathbb{W}_u^{\mathbb{P},\mathbf{b}}$ is β_{j} and η_{j}, respectively. Correspondingly, α_{j} is the sensitivity to disaster uncertainty $\sum_{\ell=\mathbb{N}_t}^{\mathbb{N}_{t+\tau}} x_\ell$. To quantitatively examine the channels that may affect currency option premiums, we consider parameterizations that allow economy j to have exposures to these uncertainties that are different from those for the United States.

The exchange rate is the ratio of the foreign to the domestic pricing kernel (for example, Backus, Foresi, and Telmer (2001)) and $\frac{F_{t,\tau}^{\text{us}|\text{j}}}{S_t^{\text{us}|\text{j}}} = \exp(\{r_t^{\text{us}} - r_t^{\text{j}}\}\tau)$. Applying Ito's lemma to equations (14)–(15) implies that

$$\begin{aligned}\log\Big(\frac{S_{t+\tau}^{\text{us}|\text{j}}}{S_{t-}^{\text{us}|\text{j}}}\Big) &= (r_t^{\text{us}} - r_t^{\text{j}})\tau + \frac{1}{2}(\beta_{\text{us}}^2 - \beta_{\text{j}}^2)\int_t^{t+\tau}\mathbf{v}_u du + \frac{1}{2}\{\eta_{\text{us}}^2 - \eta_{\text{j}}^2 - \mathbb{E}_t^{\mathbb{P}}(e^{\alpha_{\text{j}} x} - 1) + \mathbb{E}_t^{\mathbb{P}}(e^{\alpha_{\text{us}} x} - 1)\}\int_t^{t+\tau}\mathbf{b}_u\, du \\ &+ (\beta_{\text{j}} - \beta_{\text{us}})\int_t^{t+\tau}\sqrt{\mathbf{v}_u}\, d\mathbb{W}_u^{\mathbb{P},\mathbf{v}} + (\eta_{\text{j}} - \eta_{\text{us}})\int_t^{t+\tau}\sqrt{\mathbf{b}_u}\, d\mathbb{W}_u^{\mathbb{P},\mathbf{b}} + (\alpha_{\text{j}} - \alpha_{\text{us}})\sum_{\ell=\mathbb{N}_t}^{\mathbb{N}_{t+\tau}} x_\ell.\end{aligned} \tag{20}$$

The currency risk premium implied by the model in (20) is time-varying and can be of either sign. This model can be viewed as a slight modification of the one-currency models in Gabaix (2012) and Wachter (2013) and the two-currency models of Bates (1996), Carr and Wu (2007), Bakshi, Carr, and Wu (2008), Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2013), Jurek and Xu (2014), and Farhi and Gabaix (2016). Our differentiating element from extant models is the link to a theory of currency option premiums.

4.2. Risk premium of currency options

For compactness of expressions, we define the currency return (denoted by z) and option moneyness (denoted by k) for strike price K as follows:

$$z \equiv \log\left(\frac{S_{t+\tau}^{\text{us|j}}}{S_{t-}^{\text{us|j}}}\right) \in (-\infty,\infty), \qquad \text{and} \qquad k \equiv \log\left(\frac{K}{S_{t}^{\text{us|j}}}\right) \in (-\infty,\infty). \tag{21}$$

The case of OTM puts (respectively, calls) corresponds to $k < 0$ (respectively, $k > 0$). For $i = \sqrt{-1}$, we construct the risk premium on the (complex-valued) hypothetical payoff $e^{i\phi z}\{e^k - e^z\}$ as follows:

$$\mathbf{rp}_t[\phi;k] \equiv \mathbb{E}_t^{\mathbb{P}}(e^{i\phi z}\{e^k - e^z\}) - \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z}\{e^k - e^z\}), \text{ where } \phi \text{ is some parameter of the contract.} \tag{22}$$

In our setting, the risk premium on $e^{i\phi z}\{e^k - e^z\}$ is a fundamental object and relates to (moneyness-dependent) option payoffs on the downside and the upside (details in the Internet Appendix). Our treatment develops the expression for the risk premiums on $e^{i\phi z}\{e^k - e^z\}$, which is tractable for the dynamics in (20).

In what follows, we deduce general expressions for the option risk premiums. We note that $\mathfrak{Re}[\bullet]$ is the real part of the complex-valued function. All proofs are in the Internet Appendix.

Result 1 (put risk premium ($k < 0$)) *The put risk premium ($\mu^{\text{put}}_{\{t\to t+\tau\}}[k] \equiv \frac{\mathbb{E}_t^{\mathbb{P}}(\max(e^k - e^z,0))}{e^{-r_t^{\text{us}}\tau}\mathbb{E}_t^{\mathbb{Q}}(\max(e^k - e^z,0))} - e^{r_t^{\text{us}}\tau}$) is*

$$\mu^{\text{put}}_{\{t\to t+\tau\}}[k] = \underbrace{\frac{1}{e^{-r_t^{\text{us}}\tau}\mathbb{E}_t^{\mathbb{Q}}(\max(e^k - e^z,0))}}_{>0}\left(\underbrace{-\frac{1}{2}\frac{F_{t,\tau}^{\text{us|j}}}{S_t^{\text{us|j}}}\overbrace{\{\mathbb{E}_t^{\mathbb{P}}(\frac{S_{t+\tau}^{\text{us|j}}}{F_{t,\tau}^{\text{us|j}}}) - 1\}}^{\text{currency risk premium}}}_{<0 \text{ for risky currency}} - \frac{1}{\pi}\int_0^\infty \underbrace{\mathfrak{Re}[\frac{e^{-i\phi k}\,\mathbf{rp}_t[\phi;k]}{i\phi}]}_{\substack{\text{impact of higher-moment}\\ \text{risk premiums}}}\,d\phi\right). \tag{23}$$

On the downside of currency returns ($z < 0$), it can hold that $\mathbf{rp}_t[\phi;k]\big|_{k<0} > 0$. *Furthermore,*

$$\text{if } \mathbb{E}_t^{\mathbb{P}}(\frac{S_{t+\tau}^{\text{us|j}}}{F_{t,\tau}^{\text{us|j}}}) - 1 > 0, \quad \text{then the put risk premium is negative.} \tag{24}$$

$$\text{If } \mathbb{E}_t^{\mathbb{P}}(\frac{S_{t+\tau}^{\text{us|j}}}{F_{t,\tau}^{\text{us|j}}}) - 1 < 0, \quad \text{then the put risk premium } \textit{can} \text{ be } \textit{positive}. \;\blacksquare \tag{25}$$

Result 1 shows that one part of the put risk premium enters with a negative weight of $1/2$. If the currency risk premium is positive, investors pay a premium for downside protection on the reference currency. The notion of dollar safety and demand for U.S. safe assets directly affect this component of option premiums.

The contribution of $\int_0^\infty \mathfrak{Re}[\frac{e^{-i\phi k}\mathbf{rp}_t[\phi;k]}{i\phi}]\,d\phi$ in (23) is also critical. This term has a negative weight ($\frac{-1}{\pi} = -0.318$) and reflects the effect of risk premium on $e^{i\phi z}\{e^k - e^z\}$. In turn, the risk premium on currency return moments determines the risk premium on $e^{i\phi z}\{e^k - e^z\}$. Loosely seen, this interpretation arises from a series expansion; that is, $e^k - 1 + z\{i\phi(e^k-1) - 1\} + \frac{1}{2}z^2\{-\phi^2(e^k-1) - 2i\phi - 1\} + \frac{1}{6}z^3\{-i\phi^3(e^k-1) - 3i\phi - 1\} + \cdots = e^{i\phi z}\{e^k - e^z\}$.

Consistent with much of table 2, $\mu^{\text{put}}_{\{t\to t+\tau\}}[k]$ can become more negative at lower k, so deeper OTM puts elicit more negative premiums. Essentially, downside jump risk premiums are negative for risky currencies and the effect of jump risk can be more pronounced farther OTM.

Result 2 (call risk premium ($k > 0$)) *The call risk premium* ($\mu^{\text{call}}_{\{t\to t+\tau\}}[k] \equiv \frac{\mathbb{E}_t^{\mathbb{P}}(\max(e^z - e^k, 0))}{e^{-r_t^{\text{us}}\tau}\mathbb{E}_t^{\mathbb{Q}}(\max(e^z - e^k, 0))} - e^{r_t^{\text{us}}\tau}$) *is*

$$\mu^{\text{call}}_{\{t\to t+\tau\}}[k] \;=\; \frac{e^{r_t^{\text{us}}\tau}}{\mathbb{E}_t^{\mathbb{Q}}(\max(e^z - e^k, 0))}\left(\frac{1}{2}\frac{F_{t,\tau}^{\text{us|j}}}{S_t^{\text{us|j}}}\{\mathbb{E}_t^{\mathbb{P}}(\frac{S_{t+\tau}^{\text{us|j}}}{F_{t,\tau}^{\text{us|j}}}) - 1\} \;+\; \frac{1}{\pi}\int_0^\infty \mathfrak{Re}[\frac{e^{-i\phi k}\{-\mathbf{rp}_t[\phi;k]\}}{i\phi}]\,d\phi\right). \tag{26}$$

On the upside ($z > 0$), it can hold that $\{-\mathbf{rp}_t[\phi;k]\big|_{k>0}\} > 0$ *for low* $k > 0$, *whereas it can hold that*

$\{-\mathbf{rp}_t[\phi;k]\big|_{k>0}\} < 0$ *for farther OTM* $k > 0$*. Furthermore,*

$$\text{if } \mathbb{E}_t^{\mathbb{P}}\Big(\frac{S_{t+\tau}^{\text{us|j}}}{F_{t,\tau}^{\text{us|j}}}\Big) - 1 > 0, \quad \text{then} \quad \begin{cases} \text{the call risk premium can be positive for low } k > 0 \text{ and} \\ \text{the call risk premium can be negative for high } k > 0. \end{cases} \tag{27}$$

$$\text{If } \mathbb{E}_t^{\mathbb{P}}\Big(\frac{S_{t+\tau}^{\text{us|j}}}{F_{t,\tau}^{\text{us|j}}}\Big) - 1 < 0, \quad \text{then} \quad \begin{cases} \text{the call risk premium can be negative or positive for low } k > 0 \text{ and} \\ \text{the call risk premium can be negative for high } k > 0. \quad \blacksquare \end{cases} \tag{28}$$

If the currency risk premium is negative, the first term in (26) is negative, while $\int_0^\infty \mathfrak{Re}[\frac{e^{-i\phi k}\{-\mathbf{rp}_t[\phi;k]\}}{i\phi}]\,d\phi$ can get more negative for calls farther OTM. Thus, it is possible for call risk premiums to get more negative deeper OTM (for example, the Japanese yen).

Result 3 (straddle premium (volatility risk premium)) *The straddle risk premium is*

$$\mu_{\{t\to t+\tau\}}^{\text{straddle}}[k] \;=\; -\frac{1}{\frac{e^{r_t^{-\text{us}}\tau}}{2\pi}\mathbb{E}_t^{\mathbb{Q}}(\max(1-e^z,0)+\max(e^z-1,0))}\int_0^\infty \mathfrak{Re}[\frac{\mathbb{E}_t^{\mathbb{P}}(e^{i\phi z}\{1-e^z\}) \;-\; \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z}\{1-e^z\})}{i\phi}]\,d\phi. \tag{29}$$

It may be instructive to consider interpretations taking into account our empirical evidence on single-name put premiums (table 2), call premiums (table 3), straddle premiums (table 4),[10] and currency risk premiums (table 9). The notion of whether a foreign currency is risky (that is, the currency risk premium is positive) is relevant to our theory of option risk premiums.

4.3. Quantitative implications of Results 1, 2, and 3 and interpretation

How does the model fare in mimicking the observed option premiums? Working toward this goal, we develop the return characteristic functions $\mathbf{C}_t^{\mathbb{P}}[\phi] = \mathbb{E}_t^{\mathbb{P}}(e^{i\phi z})$ and $\mathbf{C}_t^{\mathbb{Q}}[\phi] = \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z})$ for the currency dynamics in (20). These expressions are displayed in equations (A4) and (A15) of the Appendix. We recognize that the associated risk premiums ($\mathbf{rp}_t[\phi;k]$ for each k) are a function of component volatility

[10]The lead term in the series expansion of $e^{i\phi z}\{1-e^z\}$ is $-z^2$; so, loosely speaking, if the risk premium on $-z^2$ is small and negative, it translates into low positive risk premium on straddles.

($\mathbf{v}_t$), disaster probability ($\mathbf{b}_t$), and the properties of the jump sizes in the pricing kernels of both economies.

Table 11 presents model-based option risk premiums. To facilitate comparisons, reported alongside are the actual sample averages. We focus on G10 currencies for compactness.

Because volatilities and disaster probabilities are latent and the pricing kernels are unobservable, we made three decisions when constructing table 11. First, the baseline parameters governing $\mathbf{v}_t$, $\mathbf{b}_t$, and jumps (that is, μ_x and σ_x) are common to all economies. Second, akin to Wachter (2013), the parameters of $\mathbf{b}_t$ dynamics are aligned with the price of a crash security (digital) on the S&P 500 equity index. Our average estimate is 0.057, and we keep $\theta_{\mathbf{b}}^{\mathbb{P}}/\kappa_{\mathbf{b}}^{\mathbb{P}} = 0.057$. Third, there is no guidance regarding the parameters of $\mathbf{v}_t$ in the presence of $\mathbf{b}_t$. Our essential criterion is whether the considered parameters yield plausible currency volatilities (for example, Brandt, Cochrane, and Santa-Clara (2006)). For example, $\sqrt{\theta_{\mathbf{v}}^{\mathbb{P}}/\kappa_{\mathbf{v}}^{\mathbb{P}}} = \sqrt{0.20/1.25} = 40\%$. Finally, we set $\beta_{\mathrm{us}} = 1$, $\eta_{\mathrm{us}} = 1$, and $\alpha_{\mathrm{us}} = 1$ for parsimony. It is important that the properties of the base economy pricing kernel be the same across economy pairs. The baseline parameters are displayed in table 11 (panel A). In our calculations, the half-life of $\mathbf{v}_t$ is longer than that of $\mathbf{b}_t$ (as reflected in $\kappa_{\mathbf{v}}^{\mathbb{P}} > \kappa_{\mathbf{b}}^{\mathbb{P}}$).

Employing the baseline parameters, we numerically search for (β_{j}, η_{j}, α_{j}) to achieve consistency with basic economy-pair features: (i) the currency volatility (table 1), (ii) the currency premium (table 9), and (iii) the interest-rate differential (table 1). Letting the data decide on the heterogeneity to global risks is compatible with, among others, Lustig, Roussanov, and Verdelhan (2011), Farhi and Gabaix (2016), Ready, Roussanov, and Ward (2017), and Colacito, Croce, Gavazzoni, and Ready (2018). The generated put, call, and straddle risk premiums are a consequence of the forms of the characteristic functions under $\mathbb{P}$ and $\mathbb{Q}$.

Our approach implies that the model can produce heterogeneity in the option premiums if two restrictions are satisfied. First, the United States needs to exhibit differential sensitivity to uncertainties with respect to other economies. This condition amounts to global shocks not canceling when determining exchange rate growth. Second, the model needs to exhibit non-normalities in the currency return distribution. This restriction enables plausible properties of the risk premiums on higher-order currency return moments.

Several noteworthy observations are based on table 11. First, if the currency premium is negative (Japan), then the put risk premium can be positive while the call risk premium can be negative. By contrast, if the currency premium is positive (New Zealand), then the put risk premium can be negative while the call risk premium can be positive. Second, the straddle premiums are comparatively smaller. Third, the model is capable of reproducing the qualitative patterns of option premiums with respect to moneyness. Table 11 (panel C), which depicts the average pattern across the G10, reinforces the evidence.

Overall, the parameterizations produce option risk premiums that are within the bootstrap confidence intervals of the sample counterparts. For example, the model-based call risk premiums are positive and less so for higher strikes for New Zealand. By contrast, the risk premiums of 10 delta puts and 10 delta calls are both negative for the euro area, whereas the model-based 25 delta call premium is positive (as manifested in the data). In this regard, we note that jump modeling is aimed at coping with distributional non-normalities and helps replicate the patterns by moneyness across single-name puts and calls.[11]

Substantial variation in α_{j} is detected. This economic mechanism supports differential impacts of pricing kernel jumps on exchange rates. Tracing this insight, $\mathbb{E}_t^{\mathbb{P}}([\alpha_{\mathrm{j}} - \alpha_{\mathrm{us}}] \sum_{\ell=\mathbb{N}_t}^{\mathbb{N}_{t+\tau}} x_\ell) = (\alpha_{\mathrm{j}} - \alpha_{\mathrm{us}}) \mu_x \mathbb{E}_t^{\mathbb{P}}(\int_t^{t+\tau} \mathbf{b}_u du)$, with $\mathbb{E}_t^{\mathbb{P}}(\int_t^{t+\tau} \mathbf{b}_u du) = \frac{\tau \theta_{\mathbf{b}}^{\mathbb{P}}}{\kappa_{\mathbf{b}}^{\mathbb{P}}} + \frac{(1-e^{-\kappa_{\mathbf{b}}^{\mathbb{P}} \tau})}{\kappa_{\mathbf{b}}^{\mathbb{P}}} (\mathbf{b}_t - \frac{\theta_{\mathbf{b}}^{\mathbb{P}}}{\kappa_{\mathbf{b}}^{\mathbb{P}}})$. The interpretation is that (absolute) deviations of α_{j} away from 1.0 can amplify the impact of jumps in high disaster states. Disparities in jump sizes — that vary across the pricing kernels — provide flexibility in calibrating the behavior of option premiums in the tails.

Describing empirical realities, the theoretical framework incorporates both small and large currency movements, as well as time-varying disaster probabilities and a bivariate depiction of stochastic currency volatility. We contribute by showing that the considered model can be consistent with the multidimensional aspects of the currency markets and risk premiums inferred from single-name currency options, and, crucially, it allows for the analyticity of option risk premiums.[12] [13]

[11] Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2013) show that disaster risk accounts for more than one-third of currency risk premiums among G10 economies. In their approach, OTM currency options convey large expected currency depreciations (appreciations) for high (low) interest-rate economies.

[12] The caveat to our analysis in table 11 is that the reported heterogeneity parameters and model-implied option risk premiums correspond to fixed delta (as reported in tables 2 and 3) and to values of ($\mathbf{v}_t$, $\mathbf{b}_t$) close to the assumed long-term means.

[13] While our focus has been on the performance of a 14 parameter model of option risk premiums, this model can be extended to allow for additive jumps in $\mathbf{v}_t$ or $\mathbf{b}_t$, or to consider alternative characterizations of jumps in the pricing kernels. There may

5. Conclusion

In this paper, we investigate how the supremacy of the U.S. dollar affects the pricing of risks — whether the downside or the upside — in the currency options market. Our empirical hypotheses are backed by a theory of currency option risk premiums and a tractable model of exchange rate dynamics with global risks as the drivers.

Our empirical exercises substantiate the following findings. First, the risk premiums on single-name currency puts are overwhelmingly negative, in essence, implying that markets are keen to insure the appreciation of the U.S. dollar. We find that 10 delta put risk premiums are typically more negative than their 25 delta counterparts.

Second, our exercises identify considerable heterogeneity in the sign of the call premiums. Crucially, these call premiums are, on average, statistically smaller than those of puts. Our evidence suggests that insurance concerns against U.S. dollar depreciations vary along the dimension of trade imbalances (more negative for economies with higher dollar revenue exposures), currency volatility, and interest-rate differentials (reflecting their membership to the set of funding or investment currencies).

Third, the excess return of single-name straddles is uniformly small in magnitude. Moreover, the straddle excess return of currency baskets is statistically insignificant. Thus, our evidence indicates that currency volatility concerns do not appear to be sizable.

Fourth, the insurance concerns reflected in option premiums are state-dependent. There are marked shifts in these premiums around periods of high uncertainty, with higher option premiums for investment currencies than for funding currencies. This evidence suggests that investors are willing to accept small negative expected option returns for the most resilient funding currencies in the bad economic states.

Fifth, our panel regressions show that economic disparity variables such as (i) yield differentials on

be room for improvement to calibrate the model to additional data dimensions and we leave these potential extensions for future research. Our evaluation, however, shows that a model that relies on economies having different exposures to global risks — and supports non-normalities in currency returns — has value in producing features of currency option returns.

five-year government bonds, (ii) quadratic variations in currency returns, and (iii) trailing currency returns can help to forecast subsequent currency option returns.

Our quantitative exercises show that option risk premiums are linked to the sign and magnitudes of the currency risk premiums and the risk premiums on higher-order currency return moments. All in all, our evidence reflects how the primacy of the U.S. dollar affects risk premiums in the currency options market.

Our theory and empirical findings can inform the search for international models in two ways. First, they impose hurdles on models that can be consistent with the data on currency option returns. Second, they point to the relevance of heterogeneity in risk exposures related to volatility, disaster probabilities, and jumps for contingent claims on currencies and option risk premiums.

References

Backus, D., Foresi, S., Telmer, C., 2001. Affine term structure models and the forward premium anomaly. Journal of Finance 56, 279–304.

Bakshi, G., Carr, P., Wu, L., 2008. Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies. Journal of Financial Economics 87, 132–156.

Bates, D., 1996. Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options. Review of Financial Studies 9, 69–107.

Brandt, M., Cochrane, J., Santa-Clara, P., 2006. International risk sharing is better than you think, or exchange rates are too smooth. Journal of Monetary Economics 53, 671–698.

Burnside, C., Eichenbaum, M., Kleshchelski, I., Rebelo, S., 2011. Do peso problems explain the returns to the carry trade?. Review of Financial Studies 24, 853–891.

Caballero, R., Farhi, E., Gourinchas, O., 2008. An equilibrium model of global imbalances and low interest rates. American Economic Review 98, 358–393.

Carr, P., Wu, L., 2007. Stochastic skew in currency options. Journal of Financial Economics 86, 213–247.

Colacito, R., Croce, M., Gavazzoni, F., Ready, R., 2018. Currency risk factors in a recursive multi-country economy. Journal of Finance 73, 2719–2756.

Daniel, K., Hodrick, R., Lu, Z., 2017. The carry trade: Risks and drawdowns. Critical Finance Review 6, 211–262.

Della Corte, P., Kozhan, R., Neuberger, A., 2021. The cross-section of currency volatility premia. Journal of Financial Economics 139, 950–970.

Della Corte, P., Sarno, L., Ramadorai, T., 2016. Volatility risk premia and exchange rate predictability. Journal of Financial Economics 120, 21–40.

Du, W., Schreger, J., 2014. Sovereign risk, currency risk, and corporate balance sheets. Working paper. Columbia University.

Farhi, E., Fraiberger, S., Gabaix, X., Ranciere, R., Verdelhan, A., 2013. Crash risk in currency markets. Working paper. Harvard and MIT.

Farhi, E., Gabaix, X., 2016. Rare disasters and exchange rates. Quarterly Journal of Economics 131, 1–52.

Farhi, E., Maggiori, M., 2018. A model of the international monetary system. Quarterly Journal of Economics 133, 295–355.

Gabaix, X., 2012. Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. Quarterly Journal of Economics 127, 645–700.

Garman, M., Kohlhagen, S., 1983. Foreign currency option values. Journal of International Money and Finance 2, 231–237.

Goldberg, L., Tille, C., 2008. Vehicle currency use in international trade. Journal of International Economics 76, 177–192.

Gopinath, G., Boz, E., Casas, C., Díez, F., Gourinchas, P.-O., Plagborg-Moller, M., 2020. Dominant currency paradigm. American Economic Review 100, 677–719.

Gopinath, G., Stein, J., 2021. Banking, trade, and the making of a dominant currency. Quarterly Journal of Economics 136, 783–830.

Gourinchas, P.-O., Rey, H., 2007. International financial adjustment. Journal of Political Economy 115, 665–703.

Hassan, T., 2013. Country size, currency unions, and international asset returns. Journal of Finance 68, 2269–2308.

Ivashina, V., Scharfstein, D., Stein, J., 2015. Dollar funding and the lending behavior of global banks. Quarterly Journal of Economics 130, 1241–1281.

Jurek, J., 2014. Crash-neutral currency carry trades. Journal of Financial Economics 113, 325–347.

Jurek, J., Xu, Z., 2014. Option-implied currency risk premia. Working paper. Princeton University.

Londono, J., Zhou, H., 2017. Variance risk premiums and the forward premium puzzle. Journal of Financial Economics 124, 415–440.

Lustig, H., Roussanov, N., Verdelhan, A., 2011. Common risk factors in currency returns. Review of Financial Studies 24, 3731–3777.

Maggiori, M., 2017. Financial intermediation, international risk sharing, and reserve currencies. American Economic Review 107, 3038–3071.

Maggiori, M., Neiman, B., Schreger, J., 2020. International currencies and capital allocation. Journal of Political Economy 128, 2019–2066.

Mancini, L., Ranaldo, A., Wrampelmeyer, J., 2013. Liquidity in the foreign exchange market: Measurement, commonality, and risk premiums. Journal of Finance 68, 1805–1841.

Mendoza, E., Quadrini, V., Rull, R., 2009. Financial integration, financial development, and global imbalances. Journal of Political Economy 117, 317–416.

Menkhoff, L., Sarno, L., Schmeling, M., Schrimpf, A., 2012. Carry trades and global foreign exchange volatility. Journal of Finance 67, 681–718.

Ready, R., Roussanov, N., Ward, C., 2017. Commodity trade and the carry trade: A tale of two countries. Journal of Finance 72, 2629–2684.

Stuart, A., Ord, K., 1987. Kendall's Advanced Theory of Statistics Vol. 1. Oxford University Press, New York.

Wachter, J., 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility?. Journal of Finance 68, 987–1035.

Wystup, U., 2006. FX Options and Structured Products. John Wiley and Sons, Chichester, UK.

Table 1

Daily data on currency option quotes with expiration maintained at 30 days

The exchange rates are expressed in U.S. dollars per unit of foreign currency; that is, the U.S. dollar is the base currency and the foreign currency is the reference. The quoted convention implies that an increase (decrease) in the exchange rate is consistent with the appreciation (depreciation) of the foreign currency. The option and forward quotes are available each day for expiration in the next 30 days. The matched daily data on the spot exchange rate, forward rate, and currency options come from a major bank. We also report the average quoted at-the-money (ATM) volatility (%, annualized), the average risk reversal of 10 delta currency options (%, annualized), and the average risk reversal of 25 delta currency options (%, annualized). These are defined as follows:

- *10 delta risk reversal*: quoted volatility of a 10 delta put minus 10 delta call.
- *25 delta risk reversal*: quoted volatility of a 25 delta put minus 25 delta call.

"Volat." is the realized monthly volatility (annualized), and $r^{\mathrm{j}} - r_t^{\mathrm{us}}$ is the interest-rate differential on 30 day deposits (annualized). Our sample contains the G10 currency pairs (the 10 largest and most liquid currencies) plus 6 additional non-G10 currency pairs.

	Base	Foreign economy j	Currency Symbol	Start date	End date	No. of options cycles	$r^{\mathrm{j}} - r_t^{\mathrm{us}}$ (%)	ATM volatility (%)	Risk reversals 10 delta (%)	Risk reversals 25 delta (%)	Volat. (%)
		Panel A: G10 economy pairs									
1	USD	Australia	AUD	1/3/2000	10/31/2019	4953	2.2	11.2	1.9	1.1	11.5
2	USD	Canada	CAD	1/3/2000	10/31/2019	4951	0.2	8.5	0.7	0.4	8.9
3	USD	Euro area	EUR	8/29/2000	10/31/2019	4790	-0.5	9.6	0.6	0.3	9.1
4	USD	Japan	JPY	1/3/2000	10/31/2019	4955	-2.1	10.0	-1.8	-1.0	9.7
5	USD	New Zealand	NZD	1/3/2000	10/31/2019	4954	2.6	12.2	2.0	1.1	12.4
6	USD	Norway	NOK	1/3/2000	10/31/2019	4946	0.9	11.4	1.1	0.6	11.6
7	USD	Sweden	SEK	1/3/2000	10/31/2019	4953	-0.3	11.3	1.0	0.6	11.2
8	USD	Switzerland	CHF	1/3/2000	10/31/2019	4953	-1.6	9.9	-0.0	-0.0	9.9
9	USD	United Kingdom	GBP	1/3/2000	10/31/2019	4953	0.5	9.0	0.9	0.5	9.1
		Panel B: Non-G10 economy pairs									
10	USD	Czech Republic	CZK	1/3/2000	10/31/2019	4950	-0.4	11.3	1.5	0.8	11.1
11	USD	Denmark	DKK	1/3/2000	10/31/2019	4956	-0.5	9.7	0.8	0.5	9.1
12	USD	Hungary	HUF	6/9/2000	10/31/2019	4818	3.6	12.9	3.3	1.8	13.2
13	USD	Korea	KRW	1/2/2002	10/31/2019	4459	0.8	9.7	2.6	1.4	8.7
14	USD	Poland	PLN	1/3/2000	10/31/2019	4936	3.2	12.8	2.4	1.3	12.5
15	USD	South Africa	ZAR	1/4/2000	10/31/2019	4946	6.4	16.5	5.1	2.8	16.6
	USD	basket: 15 currencies				73473	1.0	11.1	1.5	0.8	11.0
	USD	basket: G10				44408	0.3	10.4	0.7	0.4	10.4
	USD	basket: Non-G10				29065	2.0	12.2	2.6	1.4	11.9

Table 2
Risk premiums of currency puts

The base currency is the U.S. dollar, and the foreign currency is the reference. *Each day* we compute the excess returns of 10 delta put over the next 30 days as follows:

$$\mathbf{z}^{\text{put}}_{\{t\to t+\tau\}}[K_{10\Delta_p}] \equiv \frac{\max(K_{10\Delta_p} - S^{\text{us}|\text{j}}_{t+\tau}, 0)}{\text{put}_t[K_{10\Delta_p}]} - \exp(r^{\text{us}}_t \tau), \quad \text{and analogously for 25 delta put,}$$

where $K_{10\Delta_p}$ is the strike price of a 10 delta put, $S^{\text{us}|\text{j}}_{t+\tau}$ is the (settlement) price of the foreign currency, $\text{put}_t[K_{10\Delta_p}]$ is the put option price, and r^{us}_t is the 30-day U.S. Treasury bill rate (annualized). The table reports the sample mean, standard deviation (SD), and the 95% bootstrap confidence intervals. We use block bootstrap with 1,000 simulations. $\mathbb{1}_{\{\mathbf{z}>0\}}$ represents the proportion of excess returns that are positive. The sample period is 1/3/2000 to 10/31/2019, although data for the euro area and Hungary start in August and June 2000, respectively (see table 1). We also report the excess returns of currency baskets, which are computed by equally weighting excess returns of 10-delta (and 25-delta) puts each day t, considering all currencies, G10 currencies, and non-G10 currencies. The entries in bold indicate statistical significance of the mean excess returns according to the bootstrap confidence intervals. The reported put moneyness (in %) is calculated as, for example, $\log(\frac{K_{10\Delta_p}}{S^{\text{us}|\text{j}}_t}) < 0$.

		Panel A: 10 delta puts (farther OTM)							Panel B: 25 delta puts (nearer OTM)						
	Foreign economy (base is U.S.)	30 days Mean	SD	Bootstrap Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)	$\log(\frac{K_{10\Delta_p}}{S^{\text{us}\|\text{j}}_t})$ (%)	30 days Mean	SD	Bootstrap Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)	$\log(\frac{K_{25\Delta_p}}{S^{\text{us}\|\text{j}}_t})$ (%)
1	Australia	-0.074	4.8	-0.18	0.04	64	7	-4.7	-0.064	2.7	-0.12	0.00	26	18	-2.3
2	Canada	**-0.431**	3.8	-0.52	-0.34	85	6	-3.5	**-0.219**	2.2	-0.27	-0.16	31	18	-1.7
3	Euro area	**-0.305**	3.5	-0.38	-0.22	48	6	-3.9	**-0.184**	2.2	-0.24	-0.13	19	17	-1.9
4	Japan	0.043	4.6	-0.06	0.15	58	10	-3.7	**0.102**	2.6	0.04	0.16	22	22	-1.9
5	New Zealand	**-0.252**	3.8	-0.35	-0.16	66	7	-5.1	**-0.142**	2.3	-0.20	-0.09	26	17	-2.5
6	Norway	**-0.273**	3.6	-0.36	-0.19	62	7	-4.6	**-0.118**	2.2	-0.17	-0.07	24	19	-2.3
7	Sweden	**-0.267**	3.2	-0.34	-0.19	61	9	-4.6	-0.041	2.2	-0.09	0.01	24	21	-2.2
8	Switzerland	**-0.359**	3.0	-0.43	-0.29	34	7	-3.9	**-0.152**	2.1	-0.20	-0.11	15	20	-1.9
9	United Kingdom	-0.043	4.5	-0.15	0.06	58	7	-3.7	-0.022	2.6	-0.08	0.04	23	18	-1.8
10	Czech Republic	**-0.245**	3.9	-0.33	-0.15	51	6	-4.8	**-0.127**	2.4	-0.18	-0.07	20	18	-2.3
11	Denmark	**-0.321**	3.4	-0.40	-0.24	49	6	-4.0	**-0.164**	2.2	-0.21	-0.11	19	18	-1.9
12	Hungary	**-0.275**	3.9	-0.37	-0.19	63	6	-5.9	**-0.141**	2.5	-0.20	-0.08	26	17	-2.7
13	Korea	**-0.365**	3.4	-0.44	-0.28	50	5	-4.4	**-0.219**	2.4	-0.28	-0.16	25	15	-2.0
14	Poland	**-0.288**	3.9	-0.38	-0.21	59	6	-5.6	**-0.192**	2.4	-0.25	-0.14	23	16	-2.6
15	South Africa	**-0.315**	4.0	-0.41	-0.22	74	6	-7.6	**-0.132**	2.4	-0.19	-0.08	30	18	-3.5
	Basket: All 15	**-0.248**	2.6	-0.31	-0.19	58	16	-4.6	**-0.137**	1.6	-0.18	-0.10	23	25	-2.2
	Basket: G10	**-0.234**	2.5	-0.29	-0.17	58	17	-4.2	**-0.118**	1.6	-0.16	-0.08	23	26	-2.0
	Basket: Non-G10	**-0.309**	2.8	-0.37	-0.24	74	13	-5.4	**-0.176**	1.8	-0.22	-0.13	30	23	-2.5

Table 3
Risk premiums of currency calls

The base currency is the U.S. dollar, and the foreign currency is the reference. *Each day* we compute the excess returns of 10 delta call over the next 30 days as follows:

$$\mathbf{z}^{\text{call}}_{\{t\to t+\tau\}}[K_{10\Delta_c}] \;=\; \frac{\max(S^{\text{us}|\text{j}}_{t+\tau} - K_{10\Delta_c}, 0)}{\text{call}_t[K_{10\Delta_c}]} \;-\; \exp(r^{\text{us}}_t\tau), \quad \text{and analogously for 25 delta call,}$$

where $K_{10\Delta_c}$ is the strike price of a 10 delta call, $S^{\text{us}|\text{j}}_{t+\tau}$ is the (settlement) price of the foreign currency, $\text{call}_t[K_{10\Delta_c}]$ is the call option price, and r^{us}_t is the 30-day U.S. Treasury bill rate (annualized). The table reports the sample mean, standard deviation (SD), and the 95% bootstrap confidence intervals. We use block bootstrap with 1,000 simulations. $\mathbb{1}_{\{\mathbf{z}>0\}}$ represents the proportion of excess returns that are positive. The sample period is 1/3/2000 to 10/31/2019, although data for the euro area and Hungary start in August and June 2000, respectively (see table 1). We also report the excess returns of currency baskets, which are computed by equally weighting excess returns of 10-delta (and 25-delta) calls each day t, considering all currencies, G10 currencies, and non-G10 currencies. The entries in bold indicate statistical significance of the mean excess returns according to the bootstrap confidence intervals. The reported call moneyness (in %) is calculated as, for example, $\log(\frac{K_{10\Delta_c}}{S^{\text{us}|\text{j}}_t}) > 0$.

		Panel A: 25 delta calls (nearer OTM)							**Panel B: 10 delta calls** (farther OTM)						
	Foreign economy (base is U.S.)	30 days Mean	SD	Bootstrap Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)	$\log(\frac{K_{25\Delta_c}}{S^{\text{us}\|\text{j}}_t})$ (%)	30 days Mean	SD	Bootstrap Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)	$\log(\frac{K_{10\Delta_c}}{S^{\text{us}\|\text{j}}_t})$ (%)
1	Australia	**0.166**	2.6	0.11	0.23	14	24	2.2	0.077	4.1	-0.02	0.18	34	10	4.2
2	Canada	**-0.121**	2.3	-0.18	-0.07	18	19	1.7	**-0.278**	3.4	-0.36	-0.19	43	7	3.3
3	Euro area	0.008	2.4	-0.05	0.07	15	20	1.9	**-0.198**	3.5	-0.28	-0.11	34	8	3.8
4	Japan	**-0.278**	2.1	-0.33	-0.23	17	15	2.1	**-0.573**	2.5	-0.63	-0.51	34	4	4.5
5	New Zealand	0.181	2.5	0.12	0.25	17	24	2.4	0.005	3.8	-0.08	0.10	39	10	4.6
6	Norway	**0.085**	2.8	0.02	0.15	18	20	2.3	**0.184**	4.8	0.07	0.31	42	8	4.4
7	Sweden	-0.029	2.5	-0.09	0.03	17	19	2.3	**-0.120**	3.8	-0.20	-0.03	40	8	4.3
8	Switzerland	**0.116**	2.9	0.05	0.18	28	20	2.0	**0.142**	5.1	0.02	0.27	76	9	4.0
9	United Kingdom	-0.059	2.4	-0.11	0.00	21	20	1.8	**-0.225**	3.8	-0.31	-0.13	51	7	3.5
10	Czech Republic	**0.278**	2.9	0.21	0.35	22	23	2.3	**0.367**	5.1	0.24	0.50	61	11	4.3
11	Denmark	0.011	2.5	-0.04	0.07	16	20	1.9	**-0.130**	3.7	-0.22	-0.04	36	8	3.8
12	Hungary	**0.196**	2.8	0.13	0.26	18	22	2.5	**0.165**	4.4	0.06	0.27	42	10	4.8
13	Korea	0.039	2.9	-0.03	0.12	39	21	1.9	-0.060	5.4	-0.20	0.07	96	7	3.6
14	Poland	**0.170**	2.6	0.11	0.23	20	23	2.5	-0.017	4.1	-0.12	0.08	51	9	4.9
15	South Africa	**-0.139**	2.1	-0.19	-0.09	18	20	3.2	**-0.424**	2.9	-0.49	-0.36	44	7	6.0
	Basket: All 15	**0.079**	1.8	0.03	0.12	21	29	2.2	-0.034	2.5	-0.10	0.03	51	20	4.2
	Basket: G10	0.034	1.8	-0.01	0.08	21	29	2.1	**-0.081**	2.5	-0.14	-0.02	51	19	4.0
	Basket: Non-G10	**0.107**	2.0	0.06	0.16	18	29	2.4	-0.021	2.8	-0.09	0.05	44	18	4.6

Table 4
Risk premiums of currency straddles

The base currency is the U.S. dollar, and the foreign currency is the reference. *Each day* we compute the excess returns of currency straddle over the next 30 days as follows:

$$\mathbf{z}^{\text{straddle}}_{\{t\to t+\tau\}} = \frac{\max(S^{\text{us}|\text{j}}_{t} - S^{\text{us}|\text{j}}_{t+\tau}, 0) \;+\; \max(S^{\text{us}|\text{j}}_{t+\tau} - S^{\text{us}|\text{j}}_{t}, 0)}{\text{put}_t[S^{\text{us}|\text{j}}_{t}] \;+\; \text{call}_t[S^{\text{us}|\text{j}}_{t}]} \;-\; \exp(r^{\text{us}}_t \tau),$$

where $S^{\text{us}|\text{j}}_{t+\tau}$ is the (settlement) price of the foreign currency, $\text{put}_t[S^{\text{us}|\text{j}}_{t}]$ is the price of the ATM put option, $\text{call}_t[S^{\text{us}|\text{j}}_{t}]$ is the price of the ATM call option, and r^{us}_t is the 30-day U.S. Treasury bill rate (annualized). The table reports the sample mean, standard deviation (SD), and the 95% bootstrap confidence intervals. We use block bootstrap with 1,000 simulations. $\mathbb{1}_{\{\mathbf{z}>0\}}$ represents the proportion of excess returns that are positive. The sample period is 1/3/2000 to 10/31/2019, although data for the euro area and Hungary start in August and June 2000, respectively (see table 1). We also report the excess returns of currency baskets, which are computed by equally weighting excess returns of straddles each day t, considering all currencies, G10 currencies, and non-G10 currencies. The entries in bold indicate statistical significance of the mean excess returns according to the bootstrap confidence intervals.

		30 days		Bootstrap			
	Foreign economy (base is U.S.)	Mean	SD	Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)
1	Australia	**0.034**	0.83	0.011	0.056	5	43
2	Canada	**-0.054**	0.75	-0.075	-0.034	6	39
3	Euro area	-0.022	0.77	-0.044	0.000	4	40
4	Japan	**-0.029**	0.79	-0.050	-0.008	4	39
5	New Zealand	0.018	0.78	-0.003	0.040	5	43
6	Norway	-0.009	0.81	-0.031	0.013	5	40
7	Sweden	-0.005	0.76	-0.026	0.016	5	42
8	Switzerland	0.007	0.81	-0.015	0.030	5	41
9	United Kingdom	-0.003	0.82	-0.026	0.020	5	41
10	Czech Republic	**0.054**	0.85	0.030	0.077	4	43
11	Denmark	**-0.025**	0.77	-0.046	-0.004	4	40
12	Hungary	0.002	0.84	-0.022	0.026	6	40
13	Korea	**-0.039**	0.86	-0.065	-0.014	7	38
14	Poland	-0.002	0.82	-0.025	0.022	5	41
15	South Africa	**-0.023**	0.79	-0.045	-0.002	7	41
	Basket: All 15	-0.001	0.52	-0.017	0.015	4	38
	Basket: G10	-0.007	0.51	-0.022	0.007	4	39
	Basket: Non-G10	-0.005	0.59	-0.023	0.013	4	39

Table 5

Option risk premiums when currencies are dynamically sorted on $\log(\frac{F_{t,\tau}^{us|j}}{S_t^{us|j}})$

The base currency is the U.S. dollar. *Each day*, we assign currencies to the following three bins:

- Five economies with the *lowest* $F_{t,\tau}^{us|j}/S_t^{us|j}$ (investment currencies, i.e., those with the highest $r^j - r^{us}$)
- Five economies with medium $F_{t,\tau}^{us|j}/S_t^{us|j}$
- Five economies with the *highest* $F_{t,\tau}^{us|j}/S_t^{us|j}$ (funding currencies, i.e., those with the lowest $r^j - r^{us}$).

Then, we compute the equal-weighted excess return of currency options (panel A) and currency risk premiums (panel B). Reported are the sample mean, the standard deviation (SD), and the 95% bootstrap confidence intervals. We use block bootstrap with 1,000 simulations. The sample period is 8/3/2000 to 10/31/2019. $\mathbb{1}_{\{\mathbf{z}>0\}}$ represents the proportion of excess returns that are positive. The entries in bold indicate statistical significance of the mean excess returns according to the bootstrap confidence intervals.

		30 days		Bootstrap			
		Mean	SD	Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)
	Panel A: Currency option risk premiums (sorting currencies on $\frac{F_{t,\tau}^{us\|j}}{S_t^{us\|j}}$ **(low to high))**						
10 delta puts	High interest-rate currencies (H)	**-0.298**	3.0	-0.373	-0.223	56	13
	Medium interest-rate currencies (M)	**-0.238**	3.0	-0.317	-0.164	55	14
	Low interest-rate currencies (L)	**-0.208**	2.6	-0.271	-0.142	29	15
	High minus Low	**-0.090**	2.4	-0.151	-0.028	29	13
10 delta calls	High interest-rate currencies (H)	-0.022	2.5	-0.087	0.040	19	19
	Medium interest-rate currencies (M)	0.032	3.1	-0.046	0.107	28	17
	Low interest-rate currencies (L)	**-0.111**	2.9	-0.183	-0.042	31	15
	High minus Low	**0.090**	2.5	0.030	0.153	16	20
Straddles	High interest-rate currencies (H)	-0.006	0.56	-0.019	0.008	6	40
	Medium interest-rate currencies (M)	-0.001	0.59	-0.016	0.014	4	39
	Low interest-rate currencies (L)	0.003	0.61	-0.010	0.018	3	41
	High minus Low	-0.009	0.54	-0.023	0.005	2	50

		Annualized (%)		Bootstrap			
		Mean	SD	Lower	Upper	Max.	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)
	Panel B: Currency risk premiums (sorting currencies on $\frac{F_{t,\tau}^{us\|j}}{S_t^{us\|j}}$ **(low to high))**						
Currency	High interest-rate currencies (H)	**4.6**	11.3	3.7	5.6	14	55
	Medium interest-rate currencies (M)	**1.8**	8.8	1.1	2.5	11	53
	Low interest-rate currencies (L)	**0.8**	8.4	0.0	1.5	8	50
	High minus Low (carry strategy)	**3.8**	7.4	3.2	4.5	9	61

Table 6

Option risk premiums and *equity* volatility states: Currencies dynamically sorted on $\log(\frac{F_{t,\tau}^{\mathrm{us|j}}}{S_t^{\mathrm{us|j}}})$

The base currency is the U.S. dollar. Each day, we assign currencies to the following three bins:

- Five economies with the *lowest* $F_{t,\tau}^{\mathrm{us|j}}/S_t^{\mathrm{us|j}}$ (investment currencies, that is, those with the highest $r^{\mathrm{j}} - r^{\mathrm{us}}$)
- Five economies with medium $F_{t,\tau}^{\mathrm{us|j}}/S_t^{\mathrm{us|j}}$
- Five economies with the *highest* $F_{t,\tau}^{\mathrm{us|j}}/S_t^{\mathrm{us|j}}$ (funding currencies, that is, those with the lowest $r^{\mathrm{j}} - r^{\mathrm{us}}$).

Then, we compute the equal-weighted excess return of currency options (panel A) and currency risk premiums (panel B). Aligned with each day is also a value of the VIX equity volatility index. We divide the sample into five volatility states. Each day we compute excess return over the next 30 days on the options position. Reported are the partitioned sample mean across the volatility states and the 95% bootstrap confidence intervals. We use block bootstrap with 1,000 simulations. The sample period is 8/3/2000 to 10/31/2019. The entries in bold indicate statistical significance of the mean excess returns according to the bootstrap confidence intervals.

	$\mathrm{VIX}_t < 11$ 241 (4%)			$11 \le \mathrm{VIX}_t < 17$ 2270 (42%)			$17 \le \mathrm{VIX}_t < 24$ 1615 (30%)			$24 \le \mathrm{VIX}_t < 40$ 1102 (20%)			$\mathrm{VIX}_t \ge 40$ 189 (3%)		
		Bootstrap			Bootstrap			Bootstrap			Bootstrap			Bootstrap	
	Mean	Lower	Upper	Mean	Lower	Upper	Mean	Lower	Upper	Mean	Lower	Upper	Mean	Lower	Upper
	Panel A: 10 delta put risk premiums														
High $r^{\mathrm{j}} - r^{\mathrm{us}}$	**-0.84**	-0.91	-0.77	**-0.29**	-0.38	-0.19	**-0.36**	-0.47	-0.25	0.02	-0.28	0.36	**-0.51**	-0.78	-0.15
Medium, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**-0.85**	-0.90	-0.79	**-0.23**	-0.31	-0.16	-0.15	-0.31	0.00	-0.17	-0.42	0.13	-0.38	-0.69	0.05
Low, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**-0.83**	-0.89	-0.76	**-0.16**	-0.25	-0.07	**-0.16**	-0.28	-0.05	-0.14	-0.35	0.07	**-0.59**	-0.74	-0.42
H minus L	-0.02	-0.10	0.07	**-0.13**	-0.22	-0.03	**-0.20**	-0.30	-0.10	**0.17**	0.00	0.34	0.08	-0.20	0.37
	Panel B: 10 delta call risk premiums														
High, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**0.63**	0.32	0.96	**-0.37**	-0.43	-0.30	**0.22**	0.08	0.37	**0.38**	0.21	0.55	0.05	-0.26	0.39
Medium, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**1.35**	0.84	1.92	**-0.21**	-0.30	-0.12	**0.17**	0.01	0.32	**0.23**	0.02	0.44	**-0.57**	-0.74	-0.39
Low, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**0.89**	0.42	1.39	**-0.35**	-0.44	-0.25	0.05	-0.08	0.19	0.09	-0.08	0.28	**-0.60**	-0.73	-0.45
H minus L	-0.26	-0.64	0.10	-0.02	-0.09	0.05	**0.17**	0.04	0.31	**0.29**	0.11	0.46	**0.65**	0.33	0.96
	Panel C: Straddle risk premiums														
High, $r^{\mathrm{j}} - r^{\mathrm{us}}$	0.04	-0.02	0.08	**-0.02**	-0.04	-0.01	-0.02	-0.04	0.01	0.05	0.00	0.10	0.03	-0.04	0.11
Medium, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**0.14**	0.07	0.21	-0.01	-0.03	0.01	-0.01	-0.04	0.02	0.01	-0.03	0.06	0.01	-0.07	0.08
Low, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**0.22**	0.14	0.30	**-0.03**	-0.05	-0.01	**0.03**	0.00	0.06	0.00	-0.04	0.04	**-0.10**	-0.15	-0.06
H minus L	**-0.19**	-0.25	-0.13	0.00	-0.02	0.02	**-0.05**	-0.07	-0.02	**0.05**	0.01	0.08	**0.13**	0.07	0.19
	Panel D: Currency risk premiums														
High, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**12.2**	9.5	14.9	-0.8	-1.9	0.2	**7.2**	5.5	9.0	**13.8**	10.4	16.9	6.7	-5.4	17.4
Medium, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**9.3**	7.0	11.4	**-1.9**	-2.7	-1.0	**3.4**	2.0	4.8	**8.6**	6.1	11.1	-1.8	-11.2	7.5
Low, $r^{\mathrm{j}} - r^{\mathrm{us}}$	**12.3**	9.8	14.8	**-2.1**	-3.1	-1.3	**1.5**	0.1	2.8	**5.4**	3.4	7.6	-1.8	-8.4	4.3
H minus L	-0.1	-1.8	1.7	**1.3**	0.5	2.1	**5.8**	4.6	7.0	**8.4**	6.4	10.4	**8.6**	2.2	14.4

Table 7

Option risk premiums and *currency* volatility states: Currencies dynamically sorted on $\log(\frac{F_{t,\tau}^{\text{us}|\text{j}}}{S_t^{\text{us}|\text{j}}})$

We construct the single-name currency VIX (following the computation of dollar/euro exchange VIX (ticker: EVZ)), as follows:

$$\text{VIX}_t^{\text{fx,j}} = \mathbb{E}_t^{\mathbb{Q}}(\{-\frac{2}{\tau}\}\log(\frac{S_{t+\tau}^{\text{us}|\text{j}}}{F_{t,\tau}^{\text{us}|\text{j}}})) = \frac{2e^{r_t^{\text{us}}\tau}}{\tau}\int_0^{F_{t,\tau}^{\text{us}|\text{j}}}\frac{1}{K^2}\text{put}_t[K]dK + \frac{2e^{r\text{us}_t\tau}}{\tau}\int_{F_{t,\tau}^{\text{us}|\text{j}}}^{\infty}\frac{1}{K^2}\text{call}_t[K]dK.$$

We then build the time-series of $\overline{\text{VIX}}_t^{\text{fx}}$ as the equal-weighted average of single-name currency $\text{VIX}_t^{\text{fx,j}}$.

The base currency is the U.S. dollar. Each day, we assign currencies to the following three bins:

- Five economies with the *lowest* $F_{t,\tau}^{\text{us}|\text{j}}/S_t^{\text{us}|\text{j}}$ (investment currencies, that is, those with the highest $r^{\text{j}} - r^{\text{us}}$)
- Five economies with medium $F_{t,\tau}^{\text{us}|\text{j}}/S_t^{\text{us}|\text{j}}$
- Five economies with the *highest* $F_{t,\tau}^{\text{us}|\text{j}}/S_t^{\text{us}|\text{j}}$ (funding currencies, that is, those with the lowest $r^{\text{j}} - r^{\text{us}}$).

Then, we compute the equal-weighted excess return of currency options (panel A) and currency risk premiums (panel B). We divide the sample into five currency volatility states. Each day, we compute the excess returns over the next 30 days for the options position. Reported are the partitioned sample mean across the currency volatility states and the 95% bootstrap confidence intervals. We use block bootstrap with 1,000 simulations. The sample period is 8/3/2000 to 10/31/2019. The entries in bold indicate statistical significance of the mean excess returns according to the bootstrap confidence intervals.

	$\overline{\text{VIX}}_t^{\text{fx}} < 8$ 241 (5%)			$8 \leq \overline{\text{VIX}}_t^{\text{fx}} < 10$ 1082 (25%)			$10 \leq \overline{\text{VIX}}_t^{\text{fx}} < 12$ 1335 (30%)			$12 \leq \overline{\text{VIX}}_t^{\text{fx}} < 16$ 1332 (30%)			$\overline{\text{VIX}}_t^{\text{fx}} \geq 16$ 451 (10%)		
		Bootstrap			Bootstrap			Bootstrap			Bootstrap			Bootstrap	
	Mean	Lower	Upper	Mean	Lower	Upper	Mean	Lower	Upper	Mean	Lower	Upper	Mean	Lower	Upper
	Panel A: 10 delta put risk premiums														
High $r^{\text{j}} - r^{\text{us}}$	**0.68**	0.22	1.19	**-0.34**	-0.45	-0.23	**-0.44**	-0.53	-0.35	**-0.45**	-0.55	-0.35	0.18	-0.28	0.66
Medium $r^{\text{j}} - r^{\text{us}}$	**0.53**	0.24	0.85	**-0.47**	-0.56	-0.38	-0.08	-0.21	0.08	**-0.43**	-0.53	-0.33	-0.01	-0.41	0.45
Low $r^{\text{j}} - r^{\text{us}}$	**0.87**	0.41	1.30	**-0.19**	-0.29	-0.07	**-0.28**	-0.39	-0.17	**-0.33**	-0.43	-0.23	-0.26	-0.49	0.02
H minus L	-0.19	-0.67	0.35	**-0.15**	-0.28	-0.02	**-0.16**	-0.25	-0.09	**-0.12**	-0.20	-0.05	**0.44**	0.20	0.69
	Panel B: 10 delta call risk premiums														
High $r^{\text{j}} - r^{\text{us}}$	**0.33**	0.02	0.69	0.12	-0.03	0.25	**0.19**	0.05	0.30	**-0.27**	-0.37	-0.18	**-0.40**	-0.53	-0.28
Medium $r^{\text{j}} - r^{\text{us}}$	0.25	-0.16	0.68	**0.19**	0.05	0.35	**0.54**	0.35	0.72	**-0.44**	-0.53	-0.35	**-0.55**	-0.65	-0.44
Low $r^{\text{j}} - r^{\text{us}}$	0.20	-0.20	0.60	**-0.23**	-0.37	-0.07	**0.24**	0.10	0.39	**-0.32**	-0.43	-0.22	**-0.43**	-0.54	-0.31
H minus L	0.13	-0.13	0.39	**0.34**	0.22	0.47	-0.06	-0.20	0.08	0.05	-0.05	0.14	0.03	-0.10	0.17
	Panel C: Straddle risk premiums														
High $r^{\text{j}} - r^{\text{us}}$	**0.22**	0.14	0.29	-0.02	-0.04	0.01	**0.05**	0.02	0.07	**-0.09**	-0.11	-0.07	0.00	-0.05	0.06
Medium $r^{\text{j}} - r^{\text{us}}$	**0.28**	0.21	0.34	0.00	-0.03	0.02	**0.09**	0.06	0.12	**-0.12**	-0.15	-0.10	-0.05	-0.10	0.00
Low $r^{\text{j}} - r^{\text{us}}$	**0.28**	0.19	0.36	0.00	-0.03	0.03	**0.06**	0.03	0.09	**-0.08**	-0.10	-0.05	**-0.06**	-0.10	-0.02
H minus L	-0.06	-0.13	0.01	-0.02	-0.05	0.01	-0.01	-0.04	0.01	-0.01	-0.04	0.01	**0.06**	0.02	0.10
	Panel D: Currency risk premiums														
High $r^{\text{j}} - r^{\text{us}}$	**-4.4**	-7.3	-1.4	**2.8**	1.5	4.2	**5.7**	4.1	7.3	**6.0**	4.4	7.8	**6.5**	0.8	12.0
Medium $r^{\text{j}} - r^{\text{us}}$	**-6.7**	-9.2	-4.3	**1.0**	0.0	2.0	**4.2**	2.9	5.4	**2.0**	0.7	3.3	0.8	-3.2	4.9
Low $r^{\text{j}} - r^{\text{us}}$	**-5.2**	-7.7	-2.9	-1.0	-2.2	0.1	**3.0**	1.7	4.3	1.1	-0.2	2.3	0.9	-2.5	4.2
H minus L	**0.7**	-1.6	2.9	**3.8**	2.8	4.8	**2.7**	1.6	3.8	**4.9**	3.8	6.0	**5.6**	2.6	8.8

Table 8

Panel regressions: Macroeconomic disparities and option risk premiums

This table reports panel regressions in which the dependent variable is the excess return of (i) 10 delta puts, (ii) 10 delta calls, (iii) straddles, or (iv) currencies. We construct (economy-pair specific) macroeconomic disparity variables (all known at t) as follows:

- $r_t^{\text{j,5year}} - r_t^{\text{us,5year}}$: Interest-rate differential on a five-year government bond (source: Federal Reserve Board).
- $\text{QV}_t^{\text{fx,j}}$ (return quadratic variation of currency j): Sum of daily squared percentage changes in the spot exchange rates over the previous month.
- $\text{MA30}_t^{\text{fx,j}}$: Currency excess returns over the 30-day trailing window.
- RR10_t (Risk reversal of 10 delta options): Extracted as quoted volatility of a 10 delta put minus the 10 delta call.

The following panel regressions are performed at the end of each month:

$$\underbrace{\mathbf{z}^j_{\{t\to t+\tau\}}}_{\substack{\text{next 30 days} \\ \text{(puts,calls,straddles)}}} = \delta_0 + \delta_1\,(r_t^{\text{j,5year}} - r_t^{\text{us,5year}}) + \delta_2\,\text{QV}_t^{\text{fx,j}} + \delta_3\,\text{MA30}_t^{\text{fx,j}} + \delta_4\,\text{RR10}_t^{\text{j}} + e^{\text{j}}_{\{t,t+\tau\}} \quad \text{for } j = 1,\ldots,15 \text{ and } t = 1,\ldots,213.$$

Reported t-statistics (in square brackets) are based on robust standard errors that are clustered by currency. The markings ***, **, and * represent statistical significance at the 99%, 95%, and 90% confidence levels, respectively. There are 3,195 currency monthly observations (15 currencies and 213 months (over 02/2002 to 10/2019); some variables on Korea were not available until 02/2002). The constant is included in the panel regressions but not reported. We allow for both currency fixed effect and year fixed effects.

	10 delta put risk premiums		10 delta call risk premiums		Straddle risk premiums		Currency risk premiums	
Year fixed effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Currency fixed effect	*No*	Yes	*No*	Yes	*No*	Yes	*No*	Yes
$r^{\text{j,5year}} - r^{\text{us,5year}}$	**0.05*****	-0.04	-0.00	0.01	0.00	**-0.04****	**1.18*****	**2.88****
	[3.13]	[-0.65]	[-0.13]	[0.12]	[0.94]	[-2.56]	[6.72]	[2.39]
$\text{QV}^{\text{fx,j}} \times 100$	**-5.78****	**-6.09****	**-3.72*****	**-3.83*****	**-1.38*****	**-1.55*****	**-31.31***	-32.91
	[-2.83]	[-2.66]	[-3.34]	[-3.84]	[-3.88]	[-4.13]	[-1.92]	[-1.74]
$\text{MA30}^{\text{fx,j}}$	**-3.10****	**-3.08****	-1.25	-1.61	0.28	0.24	**-79.61*****	**-81.53*****
	[-2.51]	[-2.54]	[-0.65]	[-0.86]	[0.70]	[0.61]	[-4.34]	[-4.43]
RR10^{j}	-5.43	-6.02	**5.78***	4.66	0.73	0.23	-11.92	10.56
	[-1.66]	[-1.33]	[1.88]	[1.41]	[0.76]	[0.24]	[-0.27]	[0.17]
R^2 (%)	6	6	5	6	3	3	8	8
Within R^2 (%)	0.51	0.54	0.05	0.03	0.40	0.71	0.66	0.71

Table 9

Single-name currency risk premiums (constant horizon of 30 days)

The base currency is the U.S. dollar, and the foreign currency is the reference. *Each day*, we compute the currency excess returns on individual names as follows:

$$z^{\text{currency}}_{\{t\to t+\tau\}} \equiv \frac{S^{\text{us|j}}_{t+\tau}}{F^{\text{us|j}}_{t,\tau}} - 1, \quad \text{with} \quad \mathbb{E}^{\mathbb{P}}_t(z^{\text{currency}}_{\{t\to t+\tau\}}) \text{ defining the currency premium,}$$

where $F^{\text{us|j}}_{t,\tau}$ denotes the time-t forward rate for delivery in 30 days and $S^{\text{us|j}}_{t+\tau}$ the exchange rate recorded in 30 days from day t. The table reports annualized sample mean, standard deviation (SD), and the 95% bootstrap confidence intervals on the mean excess returns. We use block bootstrap with 1,000 simulations. We also report the skewness (Skew.) and excess kurtosis (Kurt.), both for 30-day returns. $\mathbb{1}_{\{\mathbf{z}>0\}}$ represents the proportion of currency excess returns that are positive. The final two columns show the maximum and minimum realizations. The sample period is 1/3/2000 to 10/31/2019, although data for the euro area and Hungary start in August and June 2000, respectively (see table 1). We also report the excess returns of currency baskets, which are computed by equally-weighting currency excess returns each day t, considering all currencies, G10 currencies, and non-G10 currencies. The entries in bold indicate statistical significance of the mean currency excess returns according to the bootstrap confidence intervals.

		Annualized (%)							Realization over 30 days	
				Bootstrap						
	Foreign economy (base is U.S.)	Mean	SD	Lower	Upper	$\mathbb{1}_{\{\mathbf{z}>0\}}$ (%)	Skew.	Kurt.	Max.	Min.
1	Australia	**3.1**	12.3	1.9	4.4	53	-0.6	6.6	13	-27
2	Canada	**1.0**	8.5	0.2	1.9	51	-0.5	8.0	10	-20
3	Euro area	**1.2**	9.8	0.2	2.2	50	0.1	4.4	15	-15
4	Japan	**-1.7**	9.7	-2.7	-0.8	47	0.1	3.7	14	-10
5	New Zealand	**4.5**	12.9	3.2	5.8	55	-0.1	4.5	17	-21
6	Norway	0.9	11.3	-0.2	2.0	51	-0.2	4.4	11	-20
7	Sweden	-0.4	11.4	-1.4	0.7	49	0.1	4.2	16	-19
8	Switzerland	**1.2**	10.6	0.2	2.2	50	0.2	5.1	17	-18
9	United Kingdom	-0.5	9.3	-1.3	0.4	51	-0.6	5.6	11	-16
10	Czech Republic	**2.5**	12.4	1.3	3.7	51	-0.1	4.4	20	-18
11	Denmark	0.3	9.9	-0.6	1.3	49	0.1	4.3	15	-15
12	Hungary	**4.1**	14.0	2.7	5.5	54	-0.4	5.7	17	-25
13	Korea	**2.0**	10.4	0.9	3.0	55	-0.6	10.6	20	-23
14	Poland	**4.4**	13.5	3.2	5.7	56	-0.7	6.0	16	-26
15	South Africa	**3.1**	16.5	1.5	4.8	52	-0.4	4.9	18	-28
	Basket: 15 currencies	**2.5**	9.0	1.6	3.4	53	-0.4	5.5	10	-17
	Basket: G10	**1.7**	8.2	0.9	2.4	51	-0.1	4.4	9	-15
	Basket: Non-G10	**3.2**	10.7	2.1	4.3	54	-0.6	6.6	14	-21

Table 10

Principal components of 10-delta put, 10-delta call, and straddle excess returns

Reported in this table is the fraction of the variance explained by each of the principal components (panel A). The data are daily, and we use all 15 single-name currencies and their options. Specifically, we employ the time series of (i) excess returns of 10 delta puts, (ii) excess returns of 10 delta calls, (iii) excess returns of straddles, (iv) excess return of currencies, and (v) the level of the currency VIX. The currency VIX for each single name is constructed as $\mathbb{E}_t^{\mathbb{Q}}(\{-\frac{2}{\tau}\}\log(\frac{S_{t+\tau}}{F_{t,\tau}})) = \frac{2e^{r_t^{us}\tau}}{\tau}\int_0^{F_{t,\tau}} \frac{1}{K^2}\text{put}_t[K]dK + \frac{2e^{rus_t\tau}}{\tau}\int_{F_{t,\tau}}^{\infty} \frac{1}{K^2}\text{call}_t[K]dK$. Panel B reports the loading of each economy pair on the first principal component.

Component	Excess returns of: 10 delta put	10 delta call	Straddle	Currency	Currency VIX
Panel A: Percentage of the variance explained					
1	49	41	45	63	84
2	59	51	55	72	88
3	67	59	63	78	92
4	73	66	68	82	94
5	78	71	73	85	96
6	82	76	78	88	97
7	85	81	82	91	98
8	89	85	86	93	98
9	92	89	90	95	99
10	94	92	92	96	99
11	96	95	95	98	100
12	98	97	97	99	100
13	99	98	99	99	100
14	100	100	100	100	100
15	100	100	100	100	100

	Excess returns of: 10 delta put	10 delta call	Straddle	Currency	Currency VIX
Panel B: Loadings on the first principal component					
Australia	0.27	0.26	0.24	0.27	0.27
Canada	0.33	0.36	0.35	0.30	0.27
Euro area	0.23	0.18	0.18	0.22	0.26
Japan	0.26	0.32	0.29	0.28	0.27
New Zealand	0.33	0.36	0.35	0.30	0.27
Norway	0.19	0.08	0.14	0.20	0.20
Sweden	0.20	0.20	0.26	0.25	0.26
Switzerland	0.30	0.32	0.30	0.29	0.26
United Kingdom	0.29	0.32	0.31	0.29	0.27
Czech Republic	0.04	0.13	0.11	0.09	0.23
Denmark	0.17	0.10	0.17	0.23	0.26
Hungary	0.20	0.13	0.18	0.25	0.27
Korea	0.32	0.26	0.30	0.29	0.27
Poland	0.31	0.34	0.31	0.30	0.27
South Africa	0.26	0.24	0.23	0.24	0.24

Table 11

Quantitative implications of Results 1, 2, and 3 utilizing exchange rate growth dynamics in equation (20)

Presented in this table are the findings from implementing Results 1, 2, and 3. We employ the dynamics of exchange rate growth in (20). This model allows for differential sensitivity of an economy to global risks, namely, diffusive volatility, diffusive disaster probabilities, and random jumps. The associated characteristic functions, $\mathbf{C}_t^{\mathbb{P}}[\phi]$ and $\mathbf{C}_t^{\mathbb{Q}}[\phi]$, are presented in equations (A4) and (A15) of the appendix. Our implementations parameterize the evolution of global risk variables as follows (with the U.S. dollar as the base currency):

Panel A: Parameterizations of the global variables that affect the pricing kernels

			Variance $\mathbf{v}_t$			Disaster probability $\mathbf{b}_t$			Disaster size		State Variables	
β_{us}	η_{us}	α_{us}	$\sqrt{\frac{\theta_{\mathbf{v}}^{\mathbb{P}}}{\kappa_{\mathbf{v}}^{\mathbb{P}}}}$	$\kappa_{\mathbf{v}}^{\mathbb{P}}$	$\sigma_{\mathbf{v}}$	$\frac{\theta_{\mathbf{b}}^{\mathbb{P}}}{\kappa_{\mathbf{b}}^{\mathbb{P}}}$	$\kappa_{\mathbf{b}}^{\mathbb{P}}$	$\sigma_{\mathbf{b}}$	μ_x	σ_x	$\mathbf{v}_t$	$\mathbf{b}_t$
1.0	1.00	1.0	0.40	1.25	0.40	0.057	1.05	0.25	0.1	0.1	0.20	0.06

Our parameterizations are in line with the counterparts in the literature on equity options and disasters. The currency volatility and currency risk premiums depends on the state variables $\mathbf{v}_t$ and $\mathbf{b}_t$. The reported absolute error represents the average absolute difference between the model and the data estimates of the (five) currency option premiums.

		Heterogeneity			Currency Option Risk Premiums								
		β_j	η_j	α_j	10 delta put	25 delta put	Straddle	25 delta call	10 delta call	Currency premium	Currency volatility	$r^j - r^{us}$	Absolute Error
Panel B: Single-name G10 currencies													
Australia	**Model**	1.0	0.6	1.8	-0.175	-0.137	-0.015	0.095	0.050	3.0	11.6		14%
	Data				-0.074	-0.064	0.166	0.077	-0.278	3.1	11.5	2.2	
Canada	**Model**	1.1	0.8	-0.7	-0.152	-0.090	-0.028	-0.043	-0.069	0.23	8.9		14%
	Data				-0.431	-0.219	-0.054	-0.121	-0.278	1.0	8.9	0.2	
Eurozone	**Model**	0.8	1.1	1.9	-0.213	-0.161	-0.020	0.053	-0.054	2.2	9.2		6%
	Data				-0.305	-0.184	-0.022	0.008	-0.198	1.2	9.1	-0.5	
Japan	**Model**	1.1	1.3	0.1	0.083	0.156	-0.019	-0.201	-0.257	-3.8	9.8		10%
	Data				0.043	0.102	-0.029	-0.278	-0.578	-1.7	9.7	-2.1	
New Zealand	**Model**	0.7	1.0	1.9	-0.245	-0.193	-0.020	0.140	0.075	4.6	12.4		4%
	Data				-0.252	-0.142	0.018	0.181	0.005	4.5	12.4	2.6	
Norway	**Model**	1.0	0.6	0.1	-0.187	-0.148	0.010	0.131	0.171	3.3	11.6		7%
	Data				-0.273	-0.118	-0.009	0.085	0.184	0.9	11.6	0.9	
Sweden	**Model**	0.9	1.3	-0.9	-0.146	-0.052	-0.024	-0.070	-0.094	-0.5	11.1		5%
	Data				-0.267	-0.041	-0.005	-0.029	-0.012	-0.4	11.2	-0.3	
Switzerland	**Model**	0.8	1.1	-0.5	-0.205	-0.173	-0.014	0.109	0.123	2.6	9.9		4%
	Data				-0.359	-0.152	0.007	0.007	0.116	1.2	9.9	-1.6	
United Kingdom	**Model**	0.9	1.3	0.8	-0.050	-0.007	-0.018	-0.068	-0.095	-0.7	9.4		4%
	Data				-0.043	-0.022	-0.003	-0.059	-0.225	-0.5	9.1	0.5	
Panel C: Averaged across all G10 currencies													
G10	**Model**				-0.143	-0.089	-0.016	0.016	-0.034	1.0	10.4		
	Data				-0.234	-0.118	-0.007	0.034	-0.084	1.7	10.4		

Appendix: Risk premiums on $e^{i\phi z}\{e^z - e^k\}$

In the context of Table 11, we are interested in the following calculation:

$$\mathbf{rp}_t[\phi;k] = \underbrace{\mathbb{E}_t^{\mathbb{P}}(e^{i\phi z}\{e^k - e^z\}) - \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z}\{e^k - e^z\})}_{\text{risk premium on } e^{i\phi z}\{e^k - e^z\}}, \qquad \text{for} \qquad z \equiv \underbrace{\log(S_{t+\tau}^{\text{us}|\text{j}}/S_{t-}^{\text{us}|\text{j}})}_{\text{currency return}}. \tag{A1}$$

$$= (e^k \mathbf{C}_t^{\mathbb{P}}[\phi] - \mathbf{C}_t^{\mathbb{P}}[\phi - i]) - (e^k \mathbf{C}_t^{\mathbb{Q}}[\phi] - \mathbf{C}_t^{\mathbb{Q}}[\phi - i]) \tag{A2}$$

where the characteristic functions of currency returns are defined as follows:

$$\mathbf{C}_t^{\mathbb{P}}[\phi] \equiv \mathbb{E}_t^{\mathbb{P}}(e^{i\phi \log(\frac{S_{t+\tau}}{S_{t-}})}) \qquad \text{and} \qquad \mathbf{C}_t^{\mathbb{Q}}[\phi] \equiv \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi \log(\frac{S_{t+\tau}}{S_{t-}})}), \tag{A3}$$

for the exchange rate dynamics in equation (20).

Characteristic function of $\log(\frac{S_{t+\tau}}{S_{t-}})$ **under** $\mathbb{P}$**:** We can show that (proof available from the authors)

$$\mathbf{C}_t^{\mathbb{P}}[\phi] = \mathbb{E}_t^{\mathbb{P}}(e^{i\phi \log(\frac{S_{t+\tau}}{S_{t-}})}) = \exp(i\phi\{r_t^{\text{us}} - r_t^{\text{j}}\}\tau - \mathfrak{a}^{\mathbb{P}}[\tau;\phi] - \mathfrak{b}^{\mathbb{P}}[\tau;\phi]\mathbf{v}_t - \mathfrak{e}^{\mathbb{P}}[\tau;\phi] - \mathfrak{f}^{\mathbb{P}}[\tau;\phi]\mathbf{b}_t), \tag{A4}$$

where

$$\mathfrak{b}^{\mathbb{P}}[\tau;\phi] = \frac{2c_{\mathbf{v}}^{\mathbb{P}}\{1 - e^{-\tau\delta_{\mathbf{v}}^{\mathbb{P}}}\}}{\{2\delta_{\mathbf{v}}^{\mathbb{P}} - (b_{\mathbf{v}}^{\mathbb{P}} + \delta_{\mathbf{v}}^{\mathbb{P}})\{1 - e^{-\tau\delta_{\mathbf{v}}^{\mathbb{P}}}\}\}} \quad \text{where} \quad \delta_{\mathbf{v}}^{\mathbb{P}} \equiv \sqrt{\{b_{\mathbf{v}}^{\mathbb{P}}\}^2 - 4a_{\mathbf{v}}c_{\mathbf{v}}^{\mathbb{P}}} \quad \text{and} \tag{A5}$$

$$\mathfrak{a}^{\mathbb{P}}[\tau;\phi] = -\frac{(\delta_{\mathbf{v}}^{\mathbb{P}} + b_{\mathbf{v}}^{\mathbb{P}})}{2a_{\mathbf{v}}}\theta_{\mathbf{v}}^{\mathbb{P}}\tau - \frac{\theta_{\mathbf{v}}^{\mathbb{P}}}{a_{\mathbf{v}}}\log\left(\frac{2\delta_{\mathbf{v}}^{\mathbb{P}} - (b_{\mathbf{v}}^{\mathbb{P}} + \delta_{\mathbf{v}}^{\mathbb{P}})\{1 - e^{-\tau\delta_{\mathbf{v}}^{\mathbb{P}}}\}}{2\delta_{\mathbf{v}}^{\mathbb{P}}}\right), \tag{A6}$$

with

$$a_{\mathbf{v}} \equiv -\frac{1}{2}\sigma_{\mathbf{v}}^2, \tag{A7}$$

$$b_{\mathbf{v}}^{\mathbb{P}} \equiv -\kappa_{\mathbf{v}}^{\mathbb{P}} + \sigma_{\mathbf{v}} i\phi(\beta_{\text{j}} - \beta_{\text{us}}), \;\; \text{and} \tag{A8}$$

$$c_{\mathbf{v}}^{\mathbb{P}} \equiv -\frac{1}{2}(i\phi)(i\phi - 1)\{\beta_{\text{j}} - \beta_{\text{us}}\}^2 - i\phi\{\beta_{\text{us}}^2 - \beta_{\text{us}}\beta_{\text{j}}\}. \tag{A9}$$

Additionally,

$$\mathfrak{f}^{\mathbb{P}}[\tau;\phi] = \frac{2\,c_{\mathbf{b}}^{\mathbb{P}}\{1-e^{-\tau\delta_{\mathbf{b}}^{\mathbb{P}}}\}}{\{2\,\delta_{\mathbf{b}}^{\mathbb{P}} - (b_{\mathbf{b}}^{\mathbb{P}}+\delta_{\mathbf{b}}^{\mathbb{P}})\{1-e^{-\tau\delta_{\mathbf{b}}^{\mathbb{P}}}\}\}} \quad \text{where} \quad \delta_{\mathbf{b}}^{\mathbb{P}} \equiv \sqrt{\{b_{\mathbf{b}}^{\mathbb{P}}\}^2 - 4a_{\mathbf{b}}c_{\mathbf{b}}^{\mathbb{P}}}, \tag{A10}$$

and

$$\mathfrak{e}^{\mathbb{P}}[\tau;\phi] \quad = \quad -\frac{(\delta_{\mathbf{b}}^{\mathbb{P}}+b_{\mathbf{b}}^{\mathbb{P}})}{2a_{\mathbf{b}}}\theta_{\mathbf{b}}^{\mathbb{P}}\tau - \frac{\theta_{\mathbf{b}}^{\mathbb{P}}}{a_{\mathbf{b}}}\log\left(\frac{2\delta_{\mathbf{b}}^{\mathbb{P}} - (b_{\mathbf{b}}^{\mathbb{P}}+\delta_{\mathbf{b}}^{\mathbb{P}})\{1-e^{-\tau\delta_{\mathbf{b}}^{\mathbb{P}}}\}}{2\delta_{\mathbf{b}}^{\mathbb{P}}}\right). \tag{A11}$$

Completing the solution, we set

$$a_{\mathbf{b}} \equiv -\frac{1}{2}\sigma_{\mathbf{b}}^2, \tag{A12}$$

$$b_{\mathbf{b}}^{\mathbb{P}} \equiv -\kappa_{\mathbf{b}}^{\mathbb{P}} + \sigma_{\mathbf{b}} i\phi(\eta_{\mathrm{j}} - \eta_{\mathrm{us}}), \text{ and} \tag{A13}$$

$$\begin{aligned} c_{\mathbf{b}}^{\mathbb{P}} \equiv\; & -\frac{1}{2}(i\phi)(i\phi-1)\{\eta_{\mathrm{j}}-\eta_{\mathrm{us}}\}^2 - \{\exp(i\phi\{\alpha_{\mathrm{j}}-\alpha_{\mathrm{us}}\}\mu_x + \frac{1}{2}\{i\phi(\alpha_{\mathrm{j}}-\alpha_{\mathrm{us}})\}^2\sigma_x^2) - 1\} \\ & - i\phi[\{\exp(\alpha_{\mathrm{us}}\mu_x + \frac{1}{2}\alpha_{\mathrm{us}}^2\sigma_x^2) - 1\} - \{\exp(\alpha_{\mathrm{j}}\mu_x + \frac{1}{2}\alpha_{\mathrm{j}}^2\sigma_x^2) - 1\} + \eta_{\mathrm{us}}^2 - \eta_{\mathrm{us}}\eta_{\mathrm{j}}]. \end{aligned} \tag{A14}$$

Characteristic function of $\log(\frac{S_{t+\tau}}{S_{t-}})$ **under** $\mathbb{Q}$**:** Next, we can show that

$$\mathbf{C}_t^{\mathbb{Q}}[\phi] = \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi\log(\frac{S_{t+\tau}}{S_{t-}})}) = \exp(i\phi\{r_t^{\mathrm{us}} - r_t^{\mathrm{j}}\}\tau - \mathfrak{a}^{\mathbb{Q}}[\tau;\phi] - \mathfrak{b}^{\mathbb{Q}}[\tau;\phi]\mathbf{v}_t - \mathfrak{e}^{\mathbb{Q}}[\tau;\phi] - \mathfrak{f}^{\mathbb{Q}}[\tau;\phi]\mathbf{b}_t), \tag{A15}$$

where, as before, $a_{\mathbf{v}} = -\frac{1}{2}\sigma_{\mathbf{v}}^2$,

$$\mathfrak{b}^{\mathbb{Q}}[\tau;\phi] \quad = \quad \frac{2\,c_{\mathbf{v}}^{\mathbb{Q}}\{1-e^{-\tau\delta_{\mathbf{v}}^{\mathbb{Q}}}\}}{\{2\,\delta_{\mathbf{v}}^{\mathbb{Q}} - (b_{\mathbf{v}}^{\mathbb{Q}}+\delta_{\mathbf{v}}^{\mathbb{Q}})\{1-e^{-\tau\delta_{\mathbf{v}}^{\mathbb{Q}}}\}\}} \quad \text{where} \quad \delta_{\mathbf{v}}^{\mathbb{Q}} \equiv \sqrt{\{b_{\mathbf{v}}^{\mathbb{Q}}\}^2 - 4\,a_{\mathbf{v}}\,c_{\mathbf{v}}^{\mathbb{Q}}} \text{ and} \tag{A16}$$

$$\mathfrak{a}^{\mathbb{Q}}[\tau;\phi] \quad = \quad -\frac{(\delta_{\mathbf{v}}^{\mathbb{Q}}+b_{\mathbf{v}}^{\mathbb{Q}})}{2\,a_{\mathbf{v}}}\theta_{\mathbf{v}}^{\mathbb{Q}}\,\tau - \frac{\theta_{\mathbf{v}}^{\mathbb{Q}}}{a_{\mathbf{v}}}\log\left(\frac{2\,\delta_{\mathbf{v}}^{\mathbb{Q}} - (b_{\mathbf{v}}^{\mathbb{Q}}+\delta_{\mathbf{v}}^{\mathbb{Q}})\{1-e^{-\tau\delta_{\mathbf{v}}^{\mathbb{Q}}}\}}{2\,\delta_{\mathbf{v}}^{\mathbb{Q}}}\right). \tag{A17}$$

We determine as follows:

$$b_{\mathbf{v}}^{\mathbb{Q}} = -\kappa_{\mathbf{v}}^{\mathbb{P}} + \sigma_{\mathbf{v}}\, i\phi\,(\beta_{\mathrm{j}} - \beta_{\mathrm{us}}) + \sigma_{\mathbf{v}}\beta_{\mathrm{us}} \quad \text{and} \tag{A18}$$

$$c_{\mathbf{v}}^{\mathbb{Q}} = -\frac{1}{2}(i\phi)(i\phi - 1)\beta_{\mathrm{j}}^2 - \frac{1}{2}\{-i(\phi+i)\}\{-i(\phi+i) - 1\}\beta_{\mathrm{us}}^2 - (i\phi)\{-i(\phi+i)\}\beta_{\mathrm{j}}\beta_{\mathrm{us}}. \tag{A19}$$

Additionally,

$$\mathfrak{f}^{\mathbb{Q}}[\tau;\phi] = \frac{2\,c_{\mathbf{b}}^{\mathbb{Q}}\{1 - e^{-\tau\delta_{\mathbf{b}}^{\mathbb{Q}}}\}}{\{2\,\delta_{\mathbf{b}}^{\mathbb{Q}} - (b_{\mathbf{b}}^{\mathbb{Q}} + \delta_{\mathbf{b}}^{\mathbb{Q}})\{1 - e^{-\tau\delta_{\mathbf{b}}^{\mathbb{Q}}}\}\}} \quad \text{where} \quad \delta_{\mathbf{b}}^{\mathbb{Q}} \equiv \sqrt{[b_{\mathbf{b}}^{\mathbb{Q}}]^2 - 4\,a_{\mathbf{b}}\,c_{\mathbf{b}}^{\mathbb{Q}}}, \text{ and} \tag{A20}$$

$$\mathfrak{e}^{\mathbb{Q}}[\tau;\phi] = -\frac{(\delta_{\mathbf{b}}^{\mathbb{P}} + b_{\mathbf{b}}^{\mathbb{Q}})}{2a_{\mathbf{b}}}\theta_{\mathbf{b}}^{\mathbb{Q}}\tau - \frac{\theta_{\mathbf{b}}^{\mathbb{Q}}}{a_{\mathbf{b}}}\log\left(\frac{2\,\delta_{\mathbf{b}}^{\mathbb{Q}} - (b_{\mathbf{b}}^{\mathbb{Q}} + \delta_{\mathbf{b}}^{\mathbb{Q}})\{1 - e^{-\tau\delta_{\mathbf{b}}^{\mathbb{Q}}}\}}{2\,\delta_{\mathbf{b}}^{\mathbb{Q}}}\right), \tag{A21}$$

and we set, as before, $a_{\mathbf{b}} = -\frac{1}{2}\sigma_{\mathbf{b}}^2$,

$$b_{\mathbf{b}}^{\mathbb{Q}} = -\kappa_{\mathbf{b}}^{\mathbb{P}} + \sigma_{\mathbf{b}} i\phi\{\eta_{\mathrm{j}} - \eta_{\mathrm{us}}\} + \sigma_{\mathbf{b}}\eta_{\mathrm{us}} \text{ and} \tag{A22}$$

$$\begin{aligned} c_{\mathbf{b}}^{\mathbb{Q}} = &-\frac{1}{2}(i\phi)(i\phi - 1)\eta_{\mathrm{j}}^2 + \frac{1}{2}(i\{\phi+i\})(-i\{\phi+i\} - 1)\eta_{\mathrm{us}}^2 + (i\phi)(i\{\phi+i\})\eta_{\mathrm{j}}\eta_{\mathrm{us}} \\ &-(e^{\{i\phi\alpha_{\mathrm{j}} - i\{\phi+i\}\alpha_{\mathrm{us}}\}\mu_x + \frac{1}{2}\{i\phi\alpha_{\mathrm{j}} - i\{\phi+i\}\alpha_{\mathrm{us}}\}^2\sigma_x^2} - 1) \\ &+ i\phi(e^{\alpha_{\mathrm{j}}\mu_x + \frac{1}{2}\alpha_{\mathrm{j}}^2\sigma_x^2} - 1) - i\{\phi+i\}(e^{\alpha_{\mathrm{us}}\mu_x + \frac{1}{2}\alpha_{\mathrm{us}}^2\sigma_x^2} - 1). \end{aligned} \tag{A23}$$

We have the analytical expressions corresponding to $\mathbb{E}_t^{\mathbb{P}}(e^{i\phi\log(\frac{S_{t+\tau}}{S_{t-}})})$ and $\mathbb{E}_t^{\mathbb{Q}}(e^{i\phi\log(\frac{S_{t+\tau}}{S_{t-}})})$. ■

King U.S. Dollar, Global Risks, and Currency Option Risk Premiums

Internet Appendix: Not Intended for Publication

Abstract

We provide the proof of the expression for the put risk premium and call risk premium in Section I and Section II, respectively. Section III has the proof of the expression for the straddle risk premium.

I. Proof of the risk premium of the currency put in Result 1

The strike price for the OTM put is $K < S_t^{\mathrm{us|j}}$. Recall that $z \equiv \log(\frac{S_{t+\tau}^{\mathrm{us|j}}}{S_{t-}^{\mathrm{us|j}}})$ and $k \equiv \log(\frac{K}{S_t^{\mathrm{us|j}}}) < 0$.

Let $p[z]$ and $q[z]$ denote the physical density (i.e., for $\mathbb{P}$) and the risk-neutral density (i.e., for $\mathbb{Q}$) corresponding to uncertainty z. It follows that

$$\underbrace{\mathbb{E}_t^{\mathbb{P}}(\frac{S_{t+\tau}^{\mathrm{us|j}}}{S_t^{\mathrm{us|j}}})}_{\text{time-varying}} = \int_{-\infty}^{\infty} e^z \, p[z] \, dz \qquad \text{and} \qquad \underbrace{\mathbb{E}_t^{\mathbb{Q}}(\frac{S_{t+\tau}^{\mathrm{us|j}}}{S_t^{\mathrm{us|j}}})}_{\text{time-varying}} = \int_{-\infty}^{\infty} e^z \, q[z] \, dz = \frac{F_{t,\tau}^{\mathrm{us|j}}}{S_t^{\mathrm{us|j}}}. \tag{I1}$$

Consider the risk premium of the put, $\mu_{\{t \to t+\tau\}}^{\mathrm{put}}[k] \equiv e^{r_t^{\mathrm{us}}\tau}(\frac{\mathbb{E}_t^{\mathbb{P}}(\max(e^k - e^z, 0))}{\mathbb{E}_t^{\mathbb{Q}}(\max(e^k - e^z, 0))} - 1)$. Now

$$\begin{aligned} \mathbb{E}_t^{\mathbb{P}}(\max(e^k - e^z, 0)) &= e^k \underbrace{\int_{-\infty}^{k} p[z] \, dz}_{\equiv \mathrm{Prob}^{\mathbb{P}}[z<k]} - \left(\int_{-\infty}^{\infty} e^z p[z] \, dz\right) \underbrace{\int_{-\infty}^{k} \frac{e^z p[z]}{\left(\int_{-\infty}^{\infty} e^z p[z] \, dz\right)} dz}_{\equiv \mathrm{Prob}^{\mathbb{P}^*}[z<k]} && \text{(I2)} \\ &= e^k \, \mathrm{Prob}^{\mathbb{P}}[z < k] - \mathbb{E}_t^{\mathbb{P}}(e^z) \, \mathrm{Prob}^{\mathbb{P}^*}[z < k]. && \text{(I3)} \end{aligned}$$

The measure $\mathbb{P}^*$ is defined by the Radon-Nikodym derivative $\frac{d\mathbb{P}^*}{d\mathbb{P}} = \frac{e^z p[z]}{\int_{-\infty}^{\infty} e^z p[z] dz}$. If one were to define the measure $\mathbb{Q}^*$ as $\frac{d\mathbb{Q}^*}{d\mathbb{Q}} = \frac{e^z q[z]}{\int_{-\infty}^{\infty} e^z q[z] dz}$, then the analogue under the risk-neutral probability measure is

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}}(\max(e^k - e^z, 0)) &= e^k \underbrace{\int_{-\infty}^{k} q[z] \, dz}_{\mathrm{Prob}^{\mathbb{Q}}[z<k]} - \left(\int_{-\infty}^{\infty} e^z q[z] \, dz\right) \int_{-\infty}^{k} \frac{e^z q[z]}{\left(\int_{-\infty}^{\infty} e^z q[z] \, dz\right)} dz && \text{(I4)} \\ &= e^k \, \mathrm{Prob}^{\mathbb{Q}}[z < k] - \frac{F_{t,\tau}^{\mathrm{us|j}}}{S_t^{\mathrm{us|j}}} \, \mathrm{Prob}^{\mathbb{Q}^*}[z < k]. && \text{(I5)} \end{aligned}$$

By Stuart and Ord (1987, Chapter 4), the form of the four probabilities are

$$\mathrm{Prob}^{\mathbb{P}}[z < k] = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k}\mathbb{E}_t^{\mathbb{P}}(e^{i\phi z})}{i\phi}], \qquad \mathrm{Prob}^{\mathbb{P}*}[z < k] = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k}\frac{\mathbb{E}_t^{\mathbb{P}}(e^{i\phi z + z})}{\mathbb{E}_t^{\mathbb{P}}(e^z)}}{i\phi}], \tag{I6}$$

$$\mathrm{Prob}^{\mathbb{Q}}[z < k] = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k}\mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z})}{i\phi}], \quad \text{and} \quad \mathrm{Prob}^{\mathbb{Q}*}[z < k] = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k}\frac{\mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z + z})}{\mathbb{E}_t^{\mathbb{Q}}(e^z)}}{i\phi}]. \tag{I7}$$

Aided by these expressions, we obtain the following:

$$\underbrace{\mu^{\text{put}}_{\{t\to t+\tau\}}[k]}_{\text{time-varying}} = \frac{e^{r^{\text{us}}_t\tau}}{\mathbb{E}^{\mathbb{Q}}_t(\max(e^k - e^z, 0))} \underbrace{\{\mathbb{E}^{\mathbb{P}}_t(\max(e^k - e^z, 0)) - \mathbb{E}^{\mathbb{Q}}_t(\max(e^k - e^z, 0))\}}_{\equiv\, \mathbb{A}_t}, \tag{I8}$$

where

$$\begin{aligned}\mathbb{A}_t &= e^k(\text{Prob}^{\mathbb{P}}[z<k] - \text{Prob}^{\mathbb{Q}}[z<k]) - \mathbb{E}^{\mathbb{P}}_t(e^z)\,\text{Prob}^{\mathbb{P}^*}[z<k] - \frac{F^{\text{us}|\text{j}}_{t,\tau}}{S^{\text{us}|\text{j}}_t}\text{Prob}^{\mathbb{Q}^*}[z<k]\} \quad \text{(I9)}\\ &= \underbrace{-\frac{1}{2}(\mathbb{E}^{\mathbb{P}}_t(e^z) - \frac{F^{\text{us}|\text{j}}_{t,\tau}}{S^{\text{us}|\text{j}}_t})}_{\substack{<\,0 \text{ for a risky currency}\\ \text{(time-varying)}}} - \frac{1}{\pi}\int_0^\infty \mathfrak{Re}[(\frac{\mathbb{E}^{\mathbb{P}}_t(e^{-i\phi(k-z)}\overbrace{\{e^k - e^z\}}^{>0}) - \mathbb{E}^{\mathbb{Q}}_t(e^{-i\phi(k-z)}\{e^k - e^z\})}{i\phi})]\,d\phi.\end{aligned}$$

In the region $z < k$, we sign

$$\mathbb{E}^{\mathbb{P}}_t(e^{i\phi z}\{e^k - e^z\}) \; - \; \mathbb{E}^{\mathbb{Q}}_t(e^{i\phi z}\{e^k - e^z\}) \; = \; \text{cov}^{\mathbb{Q}}_t(\frac{1}{\frac{q[z]}{p[z]}}, e^{i\phi z}\{e^k - e^z\}) \; > \; 0. \tag{I10}$$

This is because when $k > z$ and $z < 0$, then $e^{i\phi z}\{e^k - e^z\}$ is low (and positive) and $\frac{1}{\frac{q[z]}{p[z]}}$ is low.

We have shown the expression for $\mu^{\text{put}}_{\{t\to t+\tau\}}[k]$ in Result 1. ■

II. Proof of the risk premium of the currency call in Result 2

With $\mu^{\text{call}}_{\{t\to t+\tau\}}[k] \equiv e^{r^{\text{us}}_t\tau}(\frac{\mathbb{E}^{\mathbb{P}}_t(\max(e^z - e^k, 0))}{\mathbb{E}^{\mathbb{Q}}_t(\max(e^z - e^k, 0))} - 1)$ and $k \equiv \log(\frac{K}{S^{\text{us}|\text{j}}_t}) > 0$ for OTM calls, we obtain the following:

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}_t(\max(e^z - e^k, 0)) &= \left(\int_{-\infty}^{\infty} e^z p[z]\,dz\right) \underbrace{\int_k^\infty \frac{e^z p[z]}{\left(\int_{-\infty}^{\infty} e^z p[z]\,dz\right)}\,dz}_{\text{Prob}^{\mathbb{P}^*}[z>k]} \; - \; e^k \underbrace{\int_k^\infty p[z]\,dz}_{\text{Prob}^{\mathbb{P}}[z>k]} \quad \text{(I11)}\\ &= \mathbb{E}^{\mathbb{P}}_t(e^z)\,\text{Prob}^{\mathbb{P}^*}[z>k] \; - \; e^k\,\text{Prob}^{\mathbb{P}}[z>k]. \quad \text{(I12)}\end{aligned}$$

Additionally, it holds that

$$\mathbb{E}_t^{\mathbb{Q}}(\max(e^z - e^k, 0)) \;=\; \left(\int_{-\infty}^{\infty} e^z q[z]\, dz\right) \int_k^{\infty} \frac{e^z q[z]}{\left(\int_{-\infty}^{\infty} e^z q[z]\, dz\right)}\, dz \;-\; e^k \underbrace{\int_k^{\infty} p[q]\, dz}_{\text{Prob}^{\mathbb{Q}}[z>k]} \tag{I13}$$

$$= \frac{F_{t,\tau}^{\text{us|j}}}{S_t^{\text{us|j}}} \text{Prob}^{\mathbb{Q}^*}[z > k] \;-\; e^k\, \text{Prob}^{\mathbb{Q}}[z > k]. \tag{I14}$$

The form of the four probabilities are

$$\text{Prob}^{\mathbb{P}*}[z > k] = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k} \frac{\mathbb{E}_t^{\mathbb{P}}(e^{i\phi z + z})}{\mathbb{E}_t^{\mathbb{P}}(e^z)}}{i\phi}], \quad \text{Prob}^{\mathbb{P}}[z > k] = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k} \mathbb{E}_t^{\mathbb{P}}(e^{i\phi z})}{i\phi}], \tag{I15}$$

$$\text{Prob}^{\mathbb{Q}*}[z > k] = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k} \frac{\mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z + z})}{\mathbb{E}_t^{\mathbb{Q}}(e^z)}}{i\phi}], \quad \text{and} \quad \text{Prob}^{\mathbb{Q}}[z > k] = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathfrak{Re}[\frac{e^{-i\phi k} \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z})}{i\phi}]. \tag{I16}$$

Hence, we determine the following:

$$\underbrace{\mu_{\{t \to t+\tau\}}^{\text{call}}[k]}_{\text{time-varying}} \;=\; \frac{e^{r_t^{\text{us}} \tau}}{\mathbb{E}_t^{\mathbb{Q}}(\max(e^z - e^k, 0))} \underbrace{\{\mathbb{E}_t^{\mathbb{P}}(\max(e^z - e^k, 0)) - \mathbb{E}_t^{\mathbb{Q}}(\max(e^z - e^k, 0))\}}_{\equiv\, \mathbb{A}_t^{\text{up}}}, \tag{I17}$$

where

$$\mathbb{A}_t^{\text{up}} \;=\; \mathbb{E}_t^{\mathbb{P}}(e^z)\, \text{Prob}^{\mathbb{P}^*}[z > k] - \frac{F_{t,\tau}^{\text{us|j}}}{S_t^{\text{us|j}}} \text{Prob}^{\mathbb{Q}^*}[z > k] - e^k (\text{Prob}^{\mathbb{P}}[z > k] - \text{Prob}^{\mathbb{Q}}[z > k])\} \tag{I18}$$

$$= \underbrace{\frac{1}{2}(\mathbb{E}_t^{\mathbb{P}}(e^z) - \frac{F_{t,\tau}^{\text{us|j}}}{S_t^{\text{us|j}}})}_{\substack{>\,0 \text{ for a risky currency} \\ \text{(time}-\text{varying)}}} \;+\; \frac{1}{\pi} \int_0^{\infty} \mathfrak{Re}[(\frac{\mathbb{E}_t^{\mathbb{P}}(e^{-i\phi(k-z)} \overbrace{\{e^z - e^k\}}^{>0}) - \mathbb{E}_t^{\mathbb{Q}}(e^{-i\phi(k-z)} \{e^z - e^k\})}{i\phi})] d\phi.$$

In the region $z > k$, we sign

$$\mathbb{E}_t^{\mathbb{P}}(e^{i\phi z}\{e^z - e^k\}) \;-\; \mathbb{E}_t^{\mathbb{Q}}(e^{i\phi z}\{e^z - e^k\}) \;=\; \text{cov}_t^{\mathbb{Q}}(\frac{1}{\frac{q[z]}{p[z]}}, e^{i\phi z}\{e^z - e^k\}). \tag{I19}$$

When $z > k$ and $z > 0$, then $e^{i\phi z}\{e^z - e^k\}$ is high and $\frac{1}{\frac{q[z]}{p[z]}}$ is high and positive. However, corresponding to high values of k, $\frac{q[z]}{p[z]}$ can be high if $\frac{q[z]}{p[z]}$ is increasing in z. Then, $\frac{1}{\frac{q[z]}{p[z]}}$ is low. ■

III. Proof of the risk premium of the currency straddle in Result 3

Consider the straddle premium

$$\mu^{\text{straddle}}_{\{t\to t+\tau\}}[K] \equiv e^{r^{\text{us}}_t \tau}\left(\frac{\mathbb{E}^{\mathbb{P}}_t(\max(K - S^{\text{us}|\text{j}}_{t+\tau}, 0) + \max(S^{\text{us}|\text{j}}_{t+\tau} - K, 0))}{\mathbb{E}^{\mathbb{Q}}_t(\max(K - S^{\text{us}|\text{j}}_{t+\tau}, 0) + \max(S^{\text{us}|\text{j}}_{t+\tau} - K, 0))} - 1\right) \quad \text{(I20)}$$

$$= \frac{\mathbb{E}^{\mathbb{P}}_t(\max(1-e^z, 0) + \max(e^z - 1, 0)) - \mathbb{E}^{\mathbb{Q}}_t(\max(1-e^z, 0) + \max(e^z - 1, 0))}{e^{-r^{\text{us}}_t \tau}\mathbb{E}^{\mathbb{Q}}_t(\max(1-e^z, 0) + \max(e^z - 1, 0))}. \quad \text{(I21)}$$

The proof essentially combines the $\mathbb{P}$ and $\mathbb{Q}$ measure payoffs when $k = 0$. Then,

$$\text{Numerator of (I21)} = \frac{2}{\pi}\int_0^\infty \mathfrak{Re}\left[\frac{\mathbb{E}^{\mathbb{Q}}_t(e^{i\phi z})}{i\phi}\right]d\phi - \frac{2}{\pi}\int_0^\infty \mathfrak{Re}\left[\frac{\mathbb{E}^{\mathbb{P}}_t(e^{i\phi z})}{i\phi}\right]d\phi \quad \text{(I22)}$$

$$+\frac{2}{\pi}\int_0^\infty \mathfrak{Re}\left[\frac{\mathbb{E}^{\mathbb{P}}_t(e^{i\phi z + z})}{i\phi}\right]d\phi - \frac{2}{\pi}\int_0^\infty \mathfrak{Re}\left[\frac{\mathbb{E}^{\mathbb{Q}}_t(e^{i\phi z + z})}{i\phi}\right]d\phi. \quad \text{(I23)}$$

The rearrangement yields (29) of Result 3.

Conceptually the risk premiums on the (uncentered) return moments (provided they exist) enter the calculation of straddle risk premium. The power series expansion of $e^{i\phi z}\{1 - e^z\} = -z + z^2\left(-\frac{1}{2} - i\phi\right) + \frac{1}{6}z^3\left(3\phi^2 - 3i\phi - 1\right) + \frac{1}{24}iz^4\left(4\phi^3 - 6i\phi^2 - 4\phi + i\right) + \ldots$ ■